Global Analysis of Dynamic Models in Economics and Finance

Laura Gardini

Gian Italo Bischi • Carl Chiarella
Iryna Sushko
Editors

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Essays in Honour of Laura Gardini

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Editors
Prof. Dr. Gian Italo Bischi
Department of Economics, Society, Politics – DESP
University of Urbino
Urbino, Italy

Prof. Dr. Carl Chiarella
UTS Business School, Finance Discipline Group
University of Technology
Sydney, Australia

Prof. Dr. Iryna Sushko
Institute of Mathematics,
National Academy of Science of Ukraine
Kiev, Ukraine

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Preface

The MDEF (Modelli Dinamici in Economia e Finanza) Workshop has been held at the University of Urbino since 2000. The 2012 edition is particularly important as it occurs in the year of the 60th birthday of one of the conference's principal founders, Laura Gardini. In order to commemorate the occasion, a number of Laura's colleagues from around the world gladly agreed to contribute chapters to a special book dedicated to this event. This book is the outcome of that process. The book contains both a range of various applications and some articles of a mathematical and/or philosophical nature. Overall, the chapters in this book present issues in the analysis of economic and financial systems.

Urbino, Italy — Gian Italo Bischi
Sydney, NSW, Australia — Carl Chiarella
Kyiv, Ukraine — Iryna Sushko

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Introduction

This book is a collection of essays written by colleagues of Laura Gardini in honor of her 60th birthday. Most of the writers have been coauthors with Laura, and have all willingly contributed to the current book.

The book deals with a range of topics which indicate the depth of Laura's interests. New paradigms in the modelling of economic and financial systems after the recent crisis have of necessity led to the development of new mathematical methods that can grasp the complexity related to bounded rationality, heterogeneity, global interconnections, and nonlocal analysis of the economic systems under study. These developments have resulted in the consideration of evolving and adaptive systems, the global phenomena of which are related to interdependent local interactions.

Laura Gardini, currently Professor of Mathematics for Economics at the University of Urbino, has for many years been an undoubted leader and inspirer for many Italian researchers working in the field of nonlinear dynamics. Indeed, from 1998, Laura has been the Head of Research Projects within the framework of the Italian Research Project of Relevant National Interest (PRIN), such as "Dynamic Models in Economics and Finance: Evolution, Uncertainty and Forecasting", "Nonlinear Models in Economics and Finance: Complex Dynamics, Disequilibrium, Strategic Interactions", "Nonlinear Models in Economics and Finance: Interactions, Complexity and Forecasting", and "Local Interactions and Global Dynamics in Economics and Finance: Models and Tools", among others. Many important scientific results obtained by Laura and her collaborators have been presented in more than 150 publications. As a matter of fact, Laura's inspiration has spread far beyond the borders of Italy: a cursory inspection of the contents pages of the present volume will demonstrate a worldwide network of collaborators and followers of Laura.

The path of the scientific interests of Laura has not been a smooth one. After obtaining her degree in mathematics, Laura conducted applied research with the ENI Group, which is the main Italian Energy Group, where she was involved in the investigation of systems of partial differential equations and ordinary differential equations. It was during this time that she became particularly interested in the study

of dynamical systems theory. During the same period, Laura was teaching rational mechanics in the Faculty of Engineering at Ancona University.

After Laura moved to the University of Urbino as an Assistant Professor (Ricercatore), she soon became an Associate Professor (Associato) and then Full Professor (Ordinario) in Mathematics for Economics. During this period she initiated collaboration with economists on the analysis of discrete time economic dynamic models. This led to a change in her scientific interests from dynamic models in continuous time to discrete time models, that is to say to iterated maps. Indeed, time in economics, as a scientific concept, is often discontinuous, or discrete, being driven by decisions that cannot be continuously revised (a so-called decision-driven time). In her attempt to grasp the new problems and phenomena arising in the framework of discrete time, Laura came to know about the pioneering books and papers by Gumowski and Mira. A crucial step in her scientific development occurred when she started a very fruitful collaboration with Mira that has lasted for many years. In particular, Laura applied the method of critical curves proposed by Gumowski and Mira, to the global analysis of some one- and two-dimensional discrete time dynamical systems arising in economic applications. At the same time, she developed some interesting advances in the study of their properties in order to characterize global bifurcations of invariant sets. Some especially important results of Laura are related to the homoclinic bifurcations of non-invertible maps.

It is worth emphasizing that the scientific interests of Laura are not limited to models applied to economics. She has several collaborators working in other applied fields, such as electrical and mechanical engineering. Indeed, Laura is always interested in developing new ideas which may help in the study of dynamical systems in any applied field.

The scientific path of Laura through different mathematical methods and applications has allowed her to develop new approaches to study global dynamical properties by a continuous interplay between analytic, geometric, and numerical methods, very much guided by visual and intuitive approaches but always based on a general and rigorous mathematical analysis.

The contents of this volume reflect Laura's broad research interests. The initial set of articles contain many applications that illustrate how far the development of ideas that Laura was instrumental in initiating has progressed. The final five articles treat philosophical and mathematical methods.

The book opens with a discussion of heterogeneity and interactions among boundedly rational agents in the paper by Tony He. This has become a central issue in the recent finance and economics literature, and developments on the role of heterogeneous beliefs on asset pricing and the adaptive behavior of financial markets shed light into the complex behavior of financial markets and provide an explanation of some market anomalies. Tony surveys these developments and discusses the extent to which they can address the complexity, market anomalies, and stylized facts in financial markets. He shows how nonlinear heterogeneous agent models can characterize the dynamics of financial asset prices resulting from the interaction of heterogeneous agents with different attitudes towards risk

and different expectations about the future evolution of asset prices. This may explain the mechanisms leading to endogenous fluctuations of asset prices, and provide many insights into market booms and crashes, multiple market equilibria, the short-run deviation of the market price from the fundamental price and long-run convergence to the fundamental price. Moreover, combined with numerical simulations, the nonlinear heterogeneous agent models are capable of reproducing nonnormality in returns, volatility clustering, and power-law behavior.

The paper by Dieci and Westerhoff is inspired by the US housing market bubble, the dramatic collapse of which essentially triggered the recent global financial and economic crisis. However, the authors observe that significant booms and busts that occur in housing price cycles, which can be quite harmful to the real economy, have repeatedly been observed in the past. According to the literature, speculative behavior is a main driver of these price dynamics. A housing market model is proposed where part of the demand for houses is speculative, and it is shown that speculation may indeed destabilize otherwise stable housing markets. Moreover, analytical and numerical explorations indicate that there are two different routes which can lead to boom-and-bust housing price cycles. One route is via a Neimark-Sacker bifurcation, where fixed point dynamics first turn into cyclical or quasiperiodic motion and then, at least for some parameter combinations, into more complex dynamics. A second route is via a pitchfork bifurcation, where a unique steady state is accompanied by two additional steady states, so that housing markets may be permanently over- or undervalued. Also irregular switches between bull and bear markets may be observed, if certain speculative forces become stronger.

Chiarella and Di Guilmi present a survey of the literature that has grown out of the work of Hyman Minsky and, in particular, of the main models which have mathematically formalized the cyclical dynamics of a capitalist economy implied by the financial fragility hypothesis. They identify two issues that the existing literature has left unsolved. First, they propose a consistent microfoundation of the financial instability hypothesis, which involves heterogeneous agents. Second, they propose a path that leads to an integrated model of the public and the financial sectors that includes the possibility of the generation of endogenous money and credit.

Kopel and Lamantia study the global dynamics of a complementarity game with effort cost externalities. Players are assumed to adapt their effort choices over time using a time-discrete gradient adjustment process. Multiple stable equilibria may occur, and in addition, symmetric and asymmetric equilibria may coexist. Computer-assisted global analysis, together with the properties of critical curves, allows the study of the structure of the basins of attraction and their qualitative changes caused by global bifurcations as the parameters that capture the complementarity and the externality vary. For the model with nonnegative effort levels, asymmetric equilibria are more likely; hence, heterogenous effort choices emerge endogenously in the long run, despite the fact that players and games are symmetric.

The paper by Matsumoto and Szidarovszky, with its very catchy title taken from a famous Beatles song, proposes an oligopoly model where two firms, located in different countries, compete to sell homogeneous goods in a third country and receive subsidies (or taxes imposed) by their governments. The market is

characterized by an isoelastic price function, and the paper analyzes the dynamic behavior of the sequential subsidy game in which governments determine their optimal trade policies and, accordingly, the firms choose their optimal outputs. Some general results are obtained such as: when production costs are identical, then a trade policy and the corresponding optimal output are stable if the demand is elastic while multistability (i.e., coexistence of multiple attractors) and complex dynamics occur if the demand is inelastic. When the production costs are different, then a stable trade policy induces chaotic output fluctuations regardless of the demand elasticity. Policy dynamics can be chaotic if demand is elastic while multistability still occurs if the demand is inelastic.

Tonu Puu proposes an overview of models and tools that describe the emergence of economic geographical structures, a problem that has intrigued scientists since the early nineteenth century, with related questions concerning the creation of regional differences between densely and sparsely populated regions, local specialization of economic activities with resulting interregional trade, why and how such patterns change over time due to spatial flows (such as of trade or migrants) or non-material influences (such as diffusion of technical know-how, economic growth, and the phase of the business cycle). Early mathematical and classical geometric models that were set in continuous geographical space have an intuitive appeal, as the results can be easily visualized. However, many such models are so general that it is hard to use them for applications, such as prediction or planning. Further, they propose closed form solutions that can only be given in special simplified cases. To get more out of them, one would need numerical experiments, which, in principle, are greatly facilitated by the efficiency of modern computers. However, software dealing with the evolution of spatial patterns and numerical methods for partial differential equations in two dimensions is not so evolved, and ad hoc programs are needed to consider the effects of nonlinearity in light of recent developments of nonlinear dynamical systems. Here Puu is following the path set by Edward Lorenz in going from partial differential equations to ordinary differential equations and then iterative maps via Poincaré sections.

A step in the direction advocated by Puu is contained in the paper by Commendatore and Kubin, where the new economic geography approach originating from Paul Krugman's seminal contribution in 1991 is considered. In that paper, Krugman introduced the well-known Core-Periphery (CP) model which describes an economy composed of two symmetric/identical regions and two productive sectors: agriculture and manufacturing. The first is perfectly competitive, whereas in the second, the market structure is monopolistically competitive: increasing returns prevail, and factor remunerations depend upon market size. The long-term spatial distribution of the economic activities is determined by labour migration. Multiple equilibria emerge involving long-run (full or partial) agglomeration in one region or symmetric dispersion of the industrial activity across both regions. Commendatore and Kubin deal with a variant of the CP model: more specifically, they consider a symmetric footloose entrepreneur model, where the decisions to migrate of the mobile factor (entrepreneurs/human capital) involve choices among three identical regions. The model is framed in discrete time and a preliminary study

of its dynamical properties is presented. The reduced dynamical system is a two-dimensional nonlinear map. The existence of (locally stable) asymmetric equilibria is detected, which do not emerge in the two-region counterpart. The other results obtained further stress crucial differences with respect to the symmetric two-region model: Firstly, concerning the local stability analysis, the presence of a third region matters: depending on parameter values, when the manufacturing sector is absent in one of the three regions, stable asymmetric equilibria may emerge. Secondly, complex/strange two-dimensional attractors that do not exist in a two-region model, which is typically one-dimensional, are detected. Finally, a preliminary study of the global dynamics is given by exploring the self-similar structure of some basins of attraction.

Dal Forno and Merlone propose a dynamic model that describes psychological issues involved in the interaction inside small groups involved in labor management in a modern organization, but where the behavior of agents is grounded on behavioral hypotheses. Modern organizations are increasingly seen as open complex adaptive systems, with fundamental natural nonlinear structures, subject to internal and external forces that may be sources of chaos. The related existing literature focuses mainly on verbal theories where chaos is used as a metaphor. Indeed, deterministic chaos grounded on nonlinear dynamic models embedding psychological aspects of human behavior provides helpful insights in describing the complexity of small work groups. The paper not only provides a model of work group dynamics but also takes into consideration some aspects of human behavior such as motivation, injustice, and engagement. Necessary and sufficient condition for a focal equilibrium are given when an intolerant subordinate is paired with a tolerant one, whereas when both subordinates are intolerant, conditions for the stability of the equilibrium is given in which cooperation as well as conditions for the stability of the focal equilibrium to collapse. These results are obtained with two different reaction functions. Finally, the paper shows how the dynamics of work groups, where subordinates are intolerant to inequity, may be chaotic. This contribution shows how the study of dynamic models based on an interdisciplinary approach can lead to new results, thanks to the cross-disciplinary borrowing of ideas.

Tramontana and Westerhoff propose a simple financial market model with heterogeneous interacting speculators. Some believe in the persistence of bull and bear markets and thus optimistically buy if prices are high and pessimistically sell if prices are low; other speculators do the contrary, and bet on mean reversion, so that they buy if markets are undervalued and sell if they are overvalued. While some speculators are always active, other speculators only enter the market if prices are at least a certain distance away from fundamentals. Speculators are assumed to follow piecewise-linear trading rules. As the dynamics of the model are driven by a discontinuous piecewise-linear map, an analytical study of the model's behavior is possible, and the authors survey some recent results they obtained jointly with Laura. Moreover, a stochastic version of the model is discussed, where speculators randomly deviate from their core trading principles, and quite realistic dynamical behavior is numerically observed, including the emergence of bubbles and crashes, fat tails and volatility clustering.

The paper by Hommes, Sorger, and Wagener deals with an extremely important issue, namely, the possible coordination of beliefs of "boundedly rational" agents on simple linear rules in a nonlinear stochastic world, which leads to new concepts of equilibrium that generalize the "rational expectations equilibrium". This research field represents the attempt to build a new paradigm of rationality that – albeit relaxing the extremely strong assumptions of the rational expectations hypothesis – can avoid at the same time the "wilderness" implicit in the most common specifications of "boundedly rational behavior" (in which mechanical forecasting rules produce systematic mistakes). The equilibrium concept of SCEE (stochastic consistent expectations equilibrium), treated in detail in this paper, appears very promising in this respect. In an SCEE, agents' perceived law of motion (that is to say, agents' beliefs) is linear, and the implied actual law of motion is a nonlinear stochastic process with the same unconditional mean and autocorrelation structure as the perceived law of motion. The paper develops a full theoretical framework and applies this approach to the well-known OLG (overlapping generations) model.

The paper by Böhm, Chiarella, He, and Hüls investigates the impact of mean-reverting forecasts in a model of asset pricing with two groups of investors under market clearing. Fundamentalists believe that asset prices follow an exogenous stochastic process, while chartists assume that asset prices follow a stochastic geometric decay process. For high values of mean reversion, a period-doubling bifurcation occurs followed by a Neimark-Sacker bifurcation, after which homoclinic points exist inducing chaotic dynamics. Before the occurrence of homoclinic points, all orbits induce significant fluctuations with recurring symmetries and nonvanishing autocorrelations in all time series of prices and returns. After the homoclinic bifurcation, prices and returns follow alternating phases with low fluctuations near the steady state followed by phases with large excursions from the steady state (volatility clustering). This shows that nonlinearities of the deterministic model rather than random perturbations are the causes of volatility clustering and of the generation of high values of kurtosis for the long-run behavior of prices and returns (fat tails). Autocorrelations of prices and returns vanish, while those of absolute returns and squared returns persist for high-order lags. Thus, the model is able to reproduce some important empirical market features.

The paper by Abraham gives a historical overview of the main steps leading to the study of chaotic attractors and their global bifurcations. In particular, he recalls the computer-assisted methods used to obtain the sequences of global bifurcations analyzed in the book of Abraham, Gardini, and Mira "Chaos in Discrete Dynamical Systems: A Visual Introduction in 2 Dimensions" published in 1997. In that book, two special families of iterations of maps were numerically explored by very laborious computational work and manual inspection with the help of the tool of critical curves and their contact bifurcations. Abraham here considers an algorithm to automatically draw parts of the bifurcation set of families of maps. He also reports some preliminary experiments on the image entropy of the two map families studied in that book. Image entropy has been tested as a "robot mathematician" in the spirit of Stein and Ulam (in the 1950s). The motivation for the study of iterated point

mappings arising in economics and finance is also mentioned, as well as agent-based models for the social sciences.

The paper by Mira considers the embedding of a Dim1 piecewise continuous and piecewise linear map into a Dim2 invertible map. The Dim1 map belongs to the family studied by Leonov in the years 1960, the embedding being of Hénon map type. The existence domains of periodic attractors are obtained from a scanning of a parameter plane, analyzed by varying angles and senses of scanning, with independent checks of results. Such a method reliably provides the results. These results, often reformulated in terms of border collision bifurcations, have been recently applied to the study of several kinds of piecewise smooth maps arising in economics, finance, and the social sciences, as shown in several papers by Laura and her coauthors.

Piecewise differentiable maps, continuous or discontinuous, are also considered in the paper by Avrutin and Sushko, where particular bifurcation scenarios are described, characterized by border collisions and homoclinic bifurcations. They propose an overview and a classification of several bifurcation scenarios occurring in one-dimensional piecewise monotone maps, and define some basic kinds that can be observed in particular bifurcation sequences. The classification of bifurcations in smooth maps is now rather common in the literature, whereas for piecewise smooth and piecewise continuous maps, it is still at the early stages. Indeed, nonsmooth dynamical systems show many bifurcation scenarios, which are completely different to the scenarios occurring in smooth systems.

Agliari, Fournier-Prunaret, and Taha consider a class of three-dimensional maps having the property that their third iterate has separate components. These peculiar kinds of maps constitute the natural generalization of maps with decoupled second iterates, like the ones naturally obtained in best reply Cournot duopoly games, in models with expectations that expect cycles of period two or delay difference equations. Such two-dimensional systems can have periodic cycles of any period, the periodic points of which are located at the vertices of rectangles, as well as chaotic attractors of rectangular shape, that can be obtained from those of a one-dimensional map. Agliari, Fournier-Prunaret, and Taha prove similar properties for the three-dimensional generalization; however, new, and in some sense surprising, properties are shown. In fact, the local bifurcations of the cycles of the three-dimensional map prove to be of codimension 3, since at the bifurcation value three eigenvalues simultaneously cross the unit circle. To illustrate these results, an example is proposed, given by a delayed logistic map.

In the final contribution, Rosser compares natural evolution, based on selection and adaptation, with social and economic systems, evolving under the pressure of human decision making. These complex adaptive processes lead to the emergence of global hierarchies that emerge from local interactions and lead to self-organizing dynamics. The analysis of Rosser starts with the issues of hierarchy and the degree to which evolutionary processes operate at levels higher than the gene or organism, and proposes a parallel with the search for satisfactory and realistic microfoundations to macroeconomics rather than a mere assumption that the macrolevel simply mimics what goes on at the microlevel. The question is how multiple levels can emerge in

a self-organizing way from the lower levels and the role of adaptation and natural selection in this emergence. While advocates of self-organization argue that natural selection plays a central role in these processes, doubters see an invocation of mystical or even theological processes that stand aside from a proper understanding of the operation of natural selection. These debates remain unresolved, even if the trend may be favouring the advocates of a carefully developed multilevel selection approach in conjunction with a carefully formulated version of dynamic self-organization. However, it is a matter of fact that many recent economic models where economic agents are assumed to be boundedly rational and heterogeneous are often represented as myopic, interacting, adaptive, and evolving systems, leading to long-run emerging structures which are difficult to forecast on the basis of local interactions among agents.