A Multi-product Monopolist with Local Knowledge of Demand

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We study a learning model for a (quantity setting) monopolist that has incomplete knowledge of the demand:

- *n products*
- the monopolist produces a certain quantity of each commodity and observes the set of corresponding market clearing prices as well as the matrix of (cross-)price effects at those quantities.
- she estimates *n linear subjective* demand curves for her products
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In a perfectly competitive world the individual agent needs to know 'only' the relevant prices in order to choose an optimal action, in frameworks in which agents are price-maker (monopolistic competition, oligopoly and monopoly), the information set required is broader and goes from the shape of the demand curve to the possible replies of other players.
Bonanno (1990, p. 299)

Two different lines of research have investigated agents' behavior when market information is limited. The agents

a) either make decisions using a simple rule of thumb, such as the gradient rule (Baumol and Quandt, 1964; Furth,1986; Bischi and Naimzada, 1999; Puu, 1995)
b) or reconstruct the demand extrapolating information from the interaction between past decisions and market mechanisms (Negishi, 1961; Nikaido, 1975; Silvestre, 1977; Bonanno, 1990 for a survey of the Literature)

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While the second approach focuses mainly on the existence of equilibria, it fails to consider how agents coordinate in order to reach the optimal choice:

- Tuinstra (2004) analyzes a discrete time dynamic system to describe the price adjustment process within a Betrand oligopoly
- Bischi et al. (2006) propose an oligopoly game where quantity setting firms solve a profit maximization problem assuming a linear demand function and ignoring the effects of the competitors' outputs
- Naimzada and Ricchiuti (2008) analyzes a discrete time dynamic system within a monopoly framework with an homogeneous good
- Our model is a generalization of Naimzada and Ricchiuti (2008), it is richer because it allows for studying the effects of complements and substitutes on stability of the learning model.

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A quantity setting monopolist

- *n goods*
- *n objective inverse demand functions: f_i*
- substitutes: $\frac{\partial f_i}{\partial q_j} > 0$
- complements: $\frac{\partial f_i}{\partial q_j} < 0$
- a linear cost function, $C(q) = cq$, as in Silvestre (1977)
- the objective profit function is:

$$\prod(q) = \sum q^i f_i(q) - \sum c_i q^i \quad (1)$$

- and, in vector notation, profits are maximized if for all i

$$Df(q_*)^T q_* + f(q_*) - c = 0 \quad (2)$$

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- the monopolist knows the actual price, $p_{i,t}$, the related quantity demanded, $q_{i,t}$, and the slope, $f_i^{'}(q_t)$
- *n subjective demand function are estimated*
- expected profits are given by

$$\prod^{e}(q) = \sum q^i \left(f_i(q_t) + \frac{\partial f_i}{\partial q^i}(q_t)(q^i - q_t^i) \right) - \sum c_i q^i \quad (3)$$

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From the maximization of the expected profit, we get the following map:

$$q_{t+1} = (Df + Df^T)^{-1}(c + Df^T q_t - f) \tag{4}$$

Proposition 1.

There is a one-to-one relationship between steady states of the learning process and critical points of the objective profit function;

Proposition 2.
Every local minimum of the objective profit function is an unstable steady state of the learning process. Local maxima, on the other hand, may correspond to either locally stable or unstable steady states of the learning process, depending upon the curvature of the demand system;

The inverse demand function is:

$$f_i = a\exp(dq_i^{\gamma} q_j^{\delta}) + b \qquad i,j = 1,2 \qquad and \qquad i \neq j \qquad (5)$$

Hence:

$$\frac{\partial f_i}{\partial q_i} = ad\gamma\exp(dq_i^{\gamma} q_j^{\delta})q_i^{(\gamma-1)}q_j^{\delta} \qquad (6)$$

given $\gamma > 0$ the demand functions are downward-sloping if a and d have opposite signs. Moreover

$$\frac{\partial f_i}{\partial q_j} = ad\delta\exp(dq_i^{\gamma} q_j^{\delta})q_i^{(\gamma)}q_j^{\delta-1} \qquad (7)$$

Hence for $\delta < 0$ the goods are substitute while for $\delta > 0$ are complements. For $\delta = 0$ they are independent.

We have convexity of the demand functions for $a > 0$ and $d < 0$. Assuming the $b = c = 0.5$, $a = 7$ and $d = -1$ We have the following results:

1. *for* $\delta = 0$: the system is stable and converge to the max of the objective profit function (as shown in Naimzada and Ricchiuti,2008).
2. moreover the higher the complementarity, the lower the production, the higher the profits.

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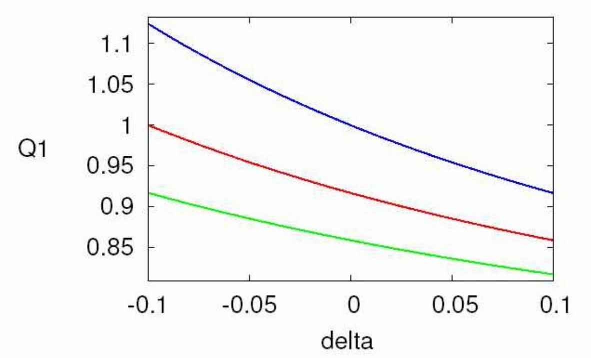
Figure: Convexity and quantity/profits: from substitute to complement



Note: $a = 7, d = -1, b = c = 0.5$, *blue* : $\gamma = 1$, *red* : $\gamma = 1.1$, *green* : $\gamma = 1.2$

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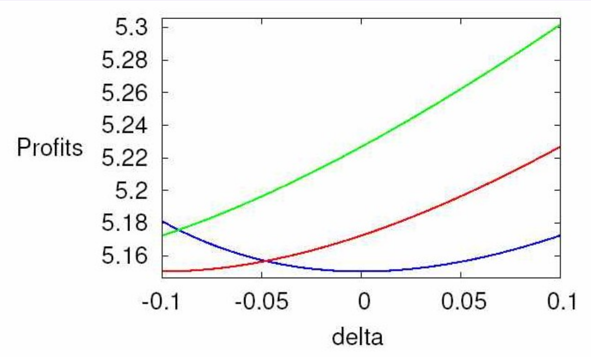

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2. *for* $\delta = 0$ *and* $a \in (-0.3, -0.01)$: route to chaos through period doubling bifurcations.

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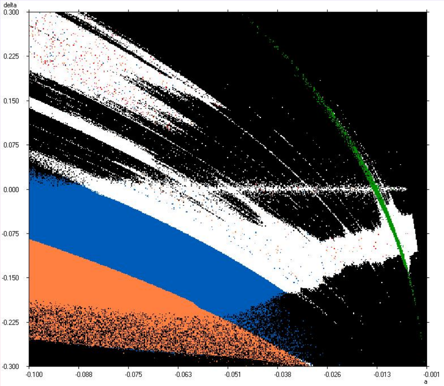
Figure: Concavity: Parameters Basin of Attraction $\delta \in (-0.3, 0 - 3) anda \in (-0.001, -0.1)$



Note: $d = 0.001, b = 7, c = 0.1, \gamma = 1, q_1 = q_2 = 1$, black=divergence, white=non convergence, red=stable,orange=period 2, green=period 3, blue=period 4 and beyond

Figure: Period 3

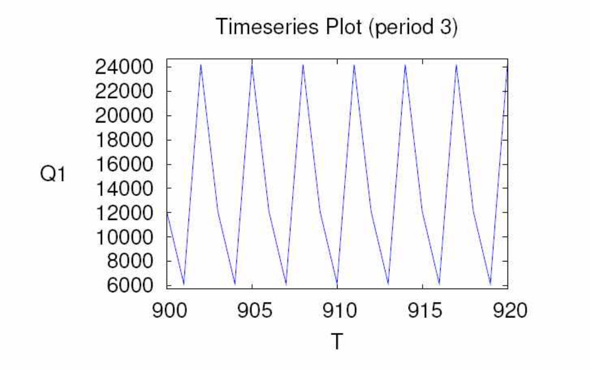


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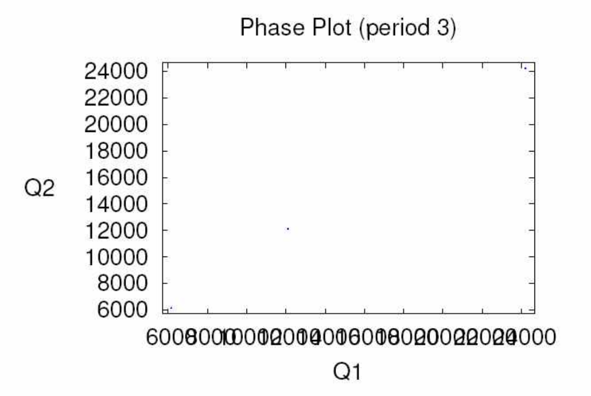
Figure: Phase plot Period 3



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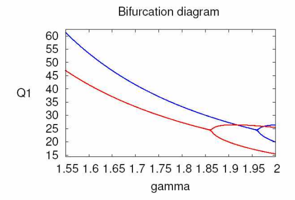
If $q_{1,t} = q_{2,t} = q_t$ the system becomes the following one-dimensional difference equation:

$$q_{t+1} = \frac{1}{2}q_t + \frac{1}{2}\frac{c - a\exp(dq_t^{(\gamma+\delta)}) - b}{ad\exp(dq_t^{(\gamma+\delta)})q_t^{(\gamma+\delta-1)}(\gamma+\delta)} \quad (8)$$

1. the diagonal is an invariant set of the system (5);
2. *for* $\gamma \in (1.5, 2)$:the higher δ the lower q_1 and a flip bifurcation is anticipated.

Figure: Bifurcation Diagram γ



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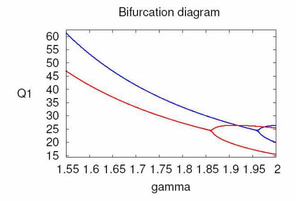

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We show that:

- As in Tuinstra (2004) and Bischi et al. (2006) there is a one-to-one relationship between steady states of the learning process and critical points of the objective profit function;
- Every local minimum of the objective profit function is an unstable steady state of the learning process. Local maxima, on the other hand, may correspond to either locally stable or unstable steady states of the learning process, depending upon the curvature of the demand system;
- By means of numerical simulations, that the learning process may lead to complicated behavior and endogenous fluctuations.

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