Financial fragility and mean-field interaction as determinants of macroeconomic dynamics: a stochastic model

Corrado Di Guilmi*, Mauro Gallegati*, Simone Landini**

*Department of Economics, Università Politecnica delle Marche, Ancona, Italy
**IRES Piemonte - Turin, Italy

5^{th} MDEF
Urbino 25/09/2008

Aim and motivation

1. To develop a proper analytical framework to address the impact of *micro-financial variables* on aggregate outcomes and apply it
2. in a dynamic model of *financial fragility* with *heterogeneous agents* using a *bottom-up* approach.

Aim and motivation

1. To develop a proper analytical framework to address the impact of *micro-financial variables* on aggregate outcomes and apply it
2. in a dynamic model of *financial fragility* with *heterogeneous agents* using a *bottom-up* approach.

Aim and motivation

1. To develop a proper analytical framework to address the impact of *micro-financial variables* on aggregate outcomes and apply it
2. in a dynamic model of *financial fragility* with *heterogeneous agents* using a *bottom-up* approach.

Outline

The approach

"Precise behavior of each agent is *irrelevant*. Rather we need to recognize that microeconomic behavior is **fundamentally stochastic** and we need to resort to proper statistical methods to study the macroeconomy consisting of a large number of such agents."
(M. Aoki and H. Yoshikawa, *Reconstructing Macroeconomics*, 2006)

Aoki's framework: stochastic dynamic modeling I

Mean-field interaction: *average interaction model that substitutes all the relations among agents that could not be analytically treated*:

- Agents are clustered in a space of **micro-states**, basing on their characteristics;
- Macro configuration is identified by the *number of agents that occupy each micro-state at a given time* (the **macro-state**), governed by a *stochastic law*;
- Modeling this stochastic law as a *continuous time Markov chain*, system's dynamics can be described by a **master equation**.

Aoki's framework: stochastic dynamic modeling I

Mean-field interaction: *average interaction model that substitutes all the relations among agents that could not be analytically treated*:

- Agents are clustered in a space of **micro-states**, basing on their characteristics;
- Macro configuration is identified by the *number of agents that occupy each micro-state at a given time* (the **macro-state**), governed by a *stochastic law*;
- Modeling this stochastic law as a *continuous time Markov chain*, system's dynamics can be described by a **master equation**.

Aoki's framework: stochastic dynamic modeling I

Mean-field interaction: *average interaction model that substitutes all the relations among agents that could not be analytically treated*:

- Agents are clustered in a space of **micro-states**, basing on their characteristics;
- Macro configuration is identified by the *number of agents that occupy each micro-state at a given time* (the **macro-state**), governed by a *stochastic law*;
- Modeling this stochastic law as a *continuous time Markov chain*, system's dynamics can be described by a **master equation**.

Aoki's framework: stochastic dynamic modeling I

Mean-field interaction: *average interaction model that substitutes all the relations among agents that could not be analytically treated*:

- Agents are clustered in a space of **micro-states**, basing on their characteristics;
- Macro configuration is identified by the *number of agents that occupy each micro-state at a given time* (the **macro-state**), governed by a *stochastic law*;
- Modeling this stochastic law as a *continuous time Markov chain*, system's dynamics can be described by a **master equation**.

Aoki's framework: stochastic dynamic modeling II

It is feasible...

- the analytical aggregation of heterogeneous agents;
- without ad-hoc hypothesis on the statistical properties of the system,

...**but**:
how to apply it?:

- implicit formulation (relationship between analytical instruments and the underlying economic model) without closed solution;

Aoki's framework: stochastic dynamic modeling II

It is feasible...

- the analytical aggregation of heterogeneous agents;
- without ad-hoc hypothesis on the statistical properties of the system,

...**but**:
how to apply it?:

- implicit formulation (relationship between analytical instruments and the underlying economic model) without closed solution;

Aoki's framework: stochastic dynamic modeling II

It is feasible...

- the analytical aggregation of heterogeneous agents;
- without ad-hoc hypothesis on the statistical properties of the system,

...**but**:
how to apply it?:

- implicit formulation (relationship between analytical instruments and the underlying economic model) without closed solution;

Aoki's framework: stochastic dynamic modeling II

It is feasible...

- the analytical aggregation of heterogeneous agents;
- without ad-hoc hypothesis on the statistical properties of the system,

...**but**:
how to apply it?:

- implicit formulation (relationship between analytical instruments and the underlying economic model) without closed solution;

Aoki's framework: stochastic dynamic modeling II

It is feasible...

- the analytical aggregation of heterogeneous agents;
- without ad-hoc hypothesis on the statistical properties of the system,

...**but**:
how to apply it?:

- implicit formulation (relationship between analytical instruments and the underlying economic model) without closed solution;

Aoki's framework: stochastic dynamic modeling II

It is feasible...

- the analytical aggregation of heterogeneous agents;
- without ad-hoc hypothesis on the statistical properties of the system,

...**but**:
how to apply it?:

- implicit formulation (relationship between analytical instruments and the underlying economic model) without closed solution;

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default ⇒ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default $\Rightarrow$ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default ⇒ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default ⇒ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default $\Rightarrow$ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default ⇒ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default $\Rightarrow$ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default $\Rightarrow$ Business cycles.

Financial fragility approach

- Minsky (1963): *Financial Instability Hypothesis* (FIH):
  - Firms are heterogeneous with respect to their financial soundness;
  - During business cycles the distribution of firms changes;
  - Financial crisis and recessions arise as the number of distressed firms increase.
- Greenwald and Stiglitz (1993) (GS): *Bankruptcy cost approach* implemented in a representative agent framework:
  - Asymmetric information and risk aversion;
  - Firms, fully rationed on equity market, recur to debt: same marginal cost for all;
  - Risk of default $\Rightarrow$ Business cycles.

Introducing heterogeneity

...**but**:

can we take into account the financial heterogeneity of firms?:

- FIH: different risk of demise $\Rightarrow$ different marginal cost of financing;
- Financial Hierarchy Hypothesis [Myers and Majluf, 1984]: different sources of financing with different marginal costs;

$\downarrow$

- assuming a given specific distribution [Gallegati, Marco, 2002]: GS with 2 classes of firms following a binomial distribution;
- computer simulations [Delli Gatti et al., 2005]: GS with heterogeneous interacting agents.

Introducing heterogeneity

...**but**:

can we take into account the financial heterogeneity of firms?:

- FIH: different risk of demise $\Rightarrow$ different marginal cost of financing;
- Financial Hierarchy Hypothesis [Myers and Majluf, 1984]: different sources of financing with different marginal costs;

$$\downarrow$$

- assuming a given specific distribution [Gallegati, Marco, 2002]: GS with 2 classes of firms following a binomial distribution;
- computer simulations [Delli Gatti et al., 2005]: GS with heterogeneous interacting agents.

Introducing heterogeneity

...**but**:

can we take into account the financial heterogeneity of firms?:

- FIH: different risk of demise ⇒ different marginal cost of financing;
- Financial Hierarchy Hypothesis [Myers and Majluf, 1984]: different sources of financing with different marginal costs;

↓

- assuming a given specific distribution [Gallegati, Marco, 2002]: GS with 2 classes of firms following a binomial distribution;
- computer simulations [Delli Gatti et al., 2005]: GS with heterogeneous interacting agents.

Introducing heterogeneity

…**but**:

can we take into account the financial heterogeneity of firms?:

- FIH: different risk of demise $\Rightarrow$ different marginal cost of financing;
- Financial Hierarchy Hypothesis [Myers and Majluf, 1984]: different sources of financing with different marginal costs;

$\downarrow$

- assuming a given specific distribution [Gallegati, Marco, 2002]: GS with 2 classes of firms following a binomial distribution;
- computer simulations [Delli Gatti et al., 2005]: GS with heterogeneous interacting agents.

Introducing heterogeneity

...**but**:

can we take into account the financial heterogeneity of firms?:

- FIH: different risk of demise ⇒ different marginal cost of financing;
- Financial Hierarchy Hypothesis [Myers and Majluf, 1984]: different sources of financing with different marginal costs;

$\downarrow$

- assuming a given specific distribution [Gallegati, Marco, 2002]: GS with 2 classes of firms following a binomial distribution;
- computer simulations [Delli Gatti et al., 2005]: GS with heterogeneous interacting agents.

Introducing heterogeneity

...**but**:

can we take into account the financial heterogeneity of firms?:

- FIH: different risk of demise ⇒ different marginal cost of financing;
- Financial Hierarchy Hypothesis [Myers and Majluf, 1984]: different sources of financing with different marginal costs;

↓

- assuming a given specific distribution [Gallegati, Marco, 2002]: GS with 2 classes of firms following a binomial distribution;
- computer simulations [Delli Gatti et al., 2005]: GS with heterogeneous interacting agents.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

↓

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

↓

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

↓

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

$\downarrow$

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

$\downarrow$

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

↓

Analytical identification of the two components of macroeconomic dynamics:

an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
    - analytical aggregation of interacting heterogeneous firms;
    - no hypothesis about micro distribution;

↓

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Results

1. A further development of the stochastic aggregation framework by employing it in
2. a dynamic model of financial fragility:
   - analytical aggregation of interacting heterogeneous firms;
   - no hypothesis about micro distribution;

$\downarrow$

Analytical identification of the two components of macroeconomic dynamics:
an ODE for *trend* and a pdf for *fluctuations*.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t)} < \mathbf{\bar{a}(t)} \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t)} \geq \mathbf{\bar{a}(t)} \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t)} < \mathbf{\bar{a}(t)} \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t)} \geq \mathbf{\bar{a}(t)} \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t)} < \mathbf{\bar{a}(t)} \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t)} \geq \mathbf{\bar{a}(t)} \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t) < \bar{a}(t)} \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t) \geq \bar{a}(t)} \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t)} < \bar{\mathbf{a}}(\mathbf{t}) \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t)} \geq \bar{\mathbf{a}}(\mathbf{t}) \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t)} < \mathbf{\bar{a}(t)} \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t)} \geq \mathbf{\bar{a}(t)} \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} state\ \mathbf{1} &: \mathbf{a_i(t)} < \bar{\mathbf{a}}(\mathbf{t}) \\ state\ \mathbf{0} &: \mathbf{a_i(t)} \geq \bar{\mathbf{a}}(\mathbf{t}) \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Assumptions

- Two microstates for firms according to their financial soundness:

$$\begin{aligned} \textit{state } \mathbf{1} &: \mathbf{a_i(t)} < \mathbf{\bar{a}(t)} \\ \textit{state } \mathbf{0} &: \mathbf{a_i(t)} \geq \mathbf{\bar{a}(t)} \end{aligned}$$

  where:
  - $a(t)$: equity ratio (own assets on total assets);
  - $\bar{a}(t)$: equity ratio for which probability of bankruptcy is 0;
- the dynamics of the number of firms in state j, N^j, follows a *continuous time jump Markov process*;
- firms fail (and exit from the system) only from state 1;
- constant number of firms N;
- bankrupted firms are immediately substituted by new ones;
- new firms entry the system in state 1.

Transition rates

Transition rates: probability to get, in a given unit of time, a "jump" of an agent from one state to another:

$$\boxed{\begin{array}{l} \lambda = \zeta\eta \\ \gamma = \iota(1-\eta) \end{array}} \tag{1}$$

where:

- ζ and ι: *transition probabilities* for a firm to move from state 0 to 1 and from 1 to 0 (*micro factor*);
- η: the a-priori probability for a firm to be in state 1 (*macro factor*).

Transition rates

Transition rates: probability to get, in a given unit of time, a "jump" of an agent from one state to another:

$$\boxed{\begin{array}{l}\lambda = \zeta\eta \\ \gamma = \iota(1-\eta)\end{array}} \qquad (1)$$

where:

- ζ and ι: *transition probabilities* for a firm to move from state 0 to 1 and from 1 to 0 (*micro factor*);
- η: the a-priori probability for a firm to be in state 1 (*macro factor*).

Transition rates

Transition rates: probability to get, in a given unit of time, a "jump" of an agent from one state to another:

$$\boxed{\begin{array}{l} \lambda = \zeta\eta \\ \gamma = \iota(1-\eta) \end{array}} \quad (1)$$

where:

- ζ and ι: *transition probabilities* for a firm to move from state 0 to 1 and from 1 to 0 (*micro factor*);
- η: the a-priori probability for a firm to be in state 1 (*macro factor*).

Transition rates

Transition rates: probability to get, in a given unit of time, a "jump" of an agent from one state to another:

$$\begin{aligned} \lambda &= \zeta\eta \\ \gamma &= \iota(1-\eta) \end{aligned} \tag{1}$$

where:

- ζ and ι: *transition probabilities* for a firm to move from state 0 to 1 and from 1 to 0 (*micro factor*);
- η: the a-priori probability for a firm to be in state 1 (*macro factor*).

Transition rates

Transition rates: probability to get, in a given unit of time, a "jump" of an agent from one state to another:

$$\boxed{\begin{aligned} \lambda &= \zeta\eta \\ \gamma &= \iota(1-\eta) \end{aligned}} \tag{1}$$

where:

- ζ and ι: *transition probabilities* for a firm to move from state 0 to 1 and from 1 to 0 (*micro factor*);
- η: the a-priori probability for a firm to be in state 1 (*macro factor*).

The system

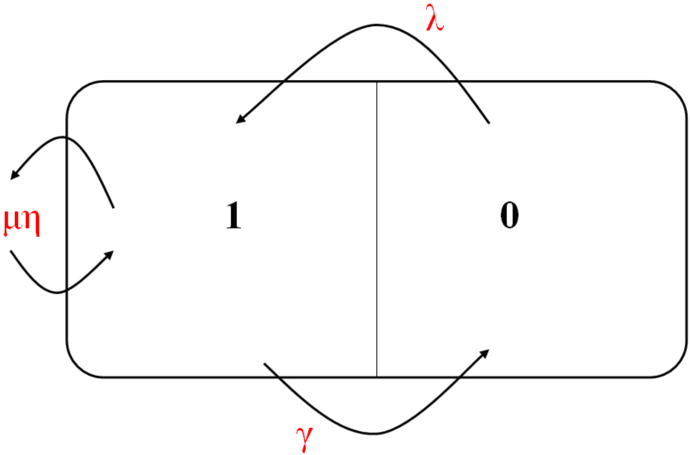

Figure: Structure of the system. μ: probability of bankruptcy for a firm.

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \quad (2)$$

  where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \tag{2}$$

where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \quad (2)$$

  where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \quad (2)$$

where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \tag{2}$$

where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \tag{2}$$

where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \tag{2}$$

  where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \tag{2}$$

  where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Assumptions

- Firms are identical within each micro-state;
- the only productive factor is capital (K);
- being fully rationed on equity market, firms recur to net worth A_i and, *if necessary*, to mortgaged debt B_i; balance sheet identity is $B_i(t) + A_i(t) = K_i(t)$;
- r is the interest rate and the return on net worth: financing costs of a firm are equal to $rK_i(t)$;
- all output is sold but each firm's selling price is affected by an iid idiosyncratic shock:

$$P_i(t) = \tilde{u}_i(t)P(t) \tag{2}$$

  where $\tilde{u}_i(t)$ has uniform distribution with $E(\tilde{u}) = 1$;
- a firm with $A \leq 0$ fails and faces bankruptcy costs equal to $C_i(t) = c(P_i(t)q_i(t))^2$;
- **Mean-field approximation** for equity ratios: a^j is a statistic of all the a_is within each state $j = 0, 1$;

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \quad (3)$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \quad (4)$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \quad (5)$$

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \tag{3}$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \tag{4}$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \tag{5}$$

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \tag{3}$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \tag{4}$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \tag{5}$$

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \tag{3}$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \tag{4}$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \tag{5}$$

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \tag{3}$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \tag{4}$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \tag{5}$$

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \tag{3}$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \tag{4}$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \tag{5}$$

Transition probabilities

Setting:

- $\bar{u}^{\zeta}(t)$ and $\bar{u}^{\iota}(t)$: the thresholds of price shock to have a switching from one state to another and
- $\bar{u}(t)$: the threshold to have bankruptcy,

we can specify transition probabilities:

$$\zeta(t) = p(\tilde{u}_i(t) \leq \bar{u}^{\zeta}) = F(\bar{u}^{\zeta}(t)) \quad (3)$$

$$\iota(t) = p(\tilde{u}_i(t) \geq \bar{u}^{\iota}) = 1 - F(\bar{u}^{\iota}(t)) \quad (4)$$

and probability of bankruptcy μ:

$$\mu(t) = p(\tilde{u}_i(t) \leq \bar{u}) = F(\bar{u}(t)) \quad (5)$$

Firms objective function

- The problem for a generic firm i is:

$$\max_{q_i(t)} \mathbb{E}\left\{P(t)\tilde{u}_i(t)q_i(t) - rK_i(t) - C_i(t)\mu(t)\right\} \qquad (6)$$

- optimal levels of production are:

$$\boxed{\begin{array}{l} q^{1*}(t) = (r + 2c\mu(t))^{-1} \\ q^{0*}(t) = r^{-1} \end{array}} \qquad (7)$$

- the aggregate output is given by:

$$Q(t) = \frac{N^1(t)}{r + 2c\mu(t)} + \frac{N^0(t)}{r} \qquad (8)$$

Firms objective function

- The problem for a generic firm i is:

$$\max_{q_i(t)} \mathbb{E}\left\{P(t)\tilde{u}_i(t)q_i(t) - rK_i(t) - C_i(t)\mu(t)\right\} \tag{6}$$

- optimal levels of production are:

$$\boxed{\begin{array}{l} q^{1*}(t) = (r + 2c\mu(t))^{-1} \\ q^{0*}(t) = r^{-1} \end{array}} \tag{7}$$

- the aggregate output is given by:

$$Q(t) = \frac{N^1(t)}{r + 2c\mu(t)} + \frac{N^0(t)}{r} \tag{8}$$

Firms objective function

- The problem for a generic firm i is:

$$\max_{q_i(t)} \mathbb{E}\left\{P(t)\tilde{u}_i(t)q_i(t) - rK_i(t) - C_i(t)\mu(t)\right\} \tag{6}$$

- optimal levels of production are:

$$\boxed{\begin{array}{l} q^{1*}(t) = (r + 2c\mu(t))^{-1} \\ q^{0*}(t) = r^{-1} \end{array}} \tag{7}$$

- the aggregate output is given by:

$$Q(t) = \frac{N^1(t)}{r + 2c\mu(t)} + \frac{N^0(t)}{r} \tag{8}$$

Stochastic analysis

Step 1: specification of the *dynamics of the joint probabilities*, in order to describe the dynamics of the macro-states; →

Step 2: analytical identification of the two components of the dynamics;

↓

Step 3: estimation of the probability η for a firm of being in state 1 (the macroscopic factor of transition rates).

Stochastic analysis

Step 1: specification of the *dynamics of the joint probabilities*, in order to describe the dynamics of the macro-states; →

Step 2: analytical identification of the two components of the dynamics;

↓

Step 3: estimation of the probability η for a firm of being in state 1 (the macroscopic factor of transition rates).

Stochastic analysis

Step 1: specification of the *dynamics of the joint probabilities*, in order to describe the dynamics of the macro-states; → Step 2: analytical identification of the two components of the dynamics;

↓

Step 3: estimation of the probability η for a firm of being in state 1 (the macroscopic factor of transition rates).

Stochastic analysis

Step 1: specification of the *dynamics of the joint probabilities*, in order to describe the dynamics of the macro-states; $\rightarrow$ Step 2: analytical identification of the two components of the dynamics;

$\downarrow$

Step 3: estimation of the probability η for a firm of being in state 1 (the macroscopic factor of transition rates).

The dynamics of macro-states

(Chapman-Kolmogorov or) **master equation**: quantifies the variation of probability flows in a small interval of time:

$$\frac{dP(N^1,t)}{dt} = \text{(inflows of probability fluxes into state 1)-(outflows of probability fluxes out of state 1)}$$

$$\frac{dP(N^1,t)}{dt} = \lambda(N-(N^1-1))P(N^1-1)+\gamma(N^1+1)P(N^1+1)+ \\ -\left[\lambda N-(\lambda-\gamma)N^1\right]P(N^1) \quad (9)$$

The dynamics of macro-states

(Chapman-Kolmogorov or) **master equation**: quantifies the variation of probability flows in a small interval of time:

$\frac{dP(N^1,t)}{dt}$ =(inflows of probability fluxes into state 1)-(outflows of probability fluxes out of state 1)

$$\frac{dP(N^1,t)}{dt} = \lambda(N-(N^1-1))P(N^1-1)+\gamma(N^1+1)P(N^1+1)+ \\ -\left[\lambda N-(\lambda-\gamma)N^1\right]P(N^1) \quad (9)$$

The dynamics of macro-states

(Chapman-Kolmogorov or) **master equation**: quantifies the variation of probability flows in a small interval of time:

$$\frac{dP(N^1,t)}{dt} = \text{(inflows of probability fluxes into state 1)-(outflows of probability fluxes out of state 1)}$$

$$\frac{dP(N^1,t)}{dt} = \lambda(N-(N^1-1))P(N^1-1)+\gamma(N^1+1)P(N^1+1)+ \\ -\left[\lambda N-(\lambda-\gamma)N^1\right]P(N^1) \tag{9}$$

Evaluating the components of the dynamics

1. split the state variable N^1 in two components:
   - the **drift** (m): tendency value of the mean for $n^1 = N^1/N$;
   - the **spread** (s): aggregate fluctuations around the drift;
   - hypothesis:

$$N^1 := Nm + \sqrt{N}s \tag{10}$$

2. Use of lead and lag operators to homogenize *in* and *out* transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Evaluating the components of the dynamics

1. split the state variable N^1 in two components:
   - the **drift** (m): tendency value of the mean for $n^1 = N^1/N$;
   - the **spread** (s): aggregate fluctuations around the drift;
   - hypothesis:

$$N^1 := Nm + \sqrt{N}s \qquad (10)$$

2. Use of lead and lag operators to homogenize *in* and *out* transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Evaluating the components of the dynamics

1. split the state variable N^1 in two components:
   - the **drift** (m): tendency value of the mean for $n^1 = N^1/N$;
   - the **spread** (s): aggregate fluctuations around the drift;
   - hypothesis:

$$N^1 := Nm + \sqrt{N}s \tag{10}$$

2. Use of lead and lag operators to homogenize *in* and *out* transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Evaluating the components of the dynamics

1. split the state variable N^1 in two components:
   - the **drift** (m): tendency value of the mean for $n^1 = N^1/N$;
   - the **spread** (s): aggregate fluctuations around the drift;
   - hypothesis:

$$N^1 := Nm + \sqrt{N}s \qquad (10)$$

2. Use of lead and lag operators to homogenize *in* and *out* transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Evaluating the components of the dynamics

1. split the state variable N^1 in two components:
   - the **drift** (m): tendency value of the mean for $n^1 = N^1/N$;
   - the **spread** (s): aggregate fluctuations around the drift;
   - hypothesis:

$$N^1 := Nm + \sqrt{N}s \quad (10)$$

2. Use of lead and lag operators to homogenize *in* and *out* transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Evaluating the components of the dynamics

1. split the state variable N^1 in two components:
   - the **drift** (m): tendency value of the mean for $n^1 = N^1/N$;
   - the **spread** (s): aggregate fluctuations around the drift;
   - hypothesis:

$$N^1 := Nm + \sqrt{N}s \tag{10}$$

2. Use of lead and lag operators to homogenize *in* and *out* transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Asymptotic solution: dynamics

- *Macroscopic equation* (the drift):

$$\frac{dm}{dt} = \lambda m - (\lambda + \gamma) m^2 \tag{11}$$

- *Fokker-Planck equation* (the spread):

$$\begin{aligned}\frac{\partial Q}{\partial t} = \; & [2(\lambda+\gamma)m - \lambda] \frac{\partial}{\partial s}(sQ(s)) + \\ & + \frac{[\lambda m(1-m) + \gamma m^2]}{2} \left(\frac{\partial}{\partial s}\right)^2 Q(s)\end{aligned} \tag{12}$$

Asymptotic solution: dynamics

- *Macroscopic equation* (the drift):

$$\frac{dm}{dt} = \lambda m - (\lambda + \gamma) m^2 \tag{11}$$

- *Fokker-Planck equation* (the spread):

$$\frac{\partial Q}{\partial t} = [2(\lambda+\gamma)m - \lambda] \frac{\partial}{\partial s}(sQ(s)) + \frac{[\lambda m(1-m) + \gamma m^2]}{2} \left(\frac{\partial}{\partial s}\right)^2 Q(s) \tag{12}$$

Asymptotic solution: dynamics

- *Macroscopic equation* (the drift):

$$\frac{dm}{dt} = \lambda m - (\lambda + \gamma) m^2 \tag{11}$$

- *Fokker-Planck equation* (the spread):

$$\begin{aligned}\frac{\partial Q}{\partial t} = \quad & [2(\lambda + \gamma)m - \lambda] \frac{\partial}{\partial s}(sQ(s)) + \\ & + \frac{\left[\lambda m(1-m) + \gamma m^2\right]}{2} \left(\frac{\partial}{\partial s}\right)^2 Q(s)\end{aligned} \tag{12}$$

Asymptotic solutions: stationary equilibrium

- Trend dynamics and stationary state:

$$m(t) = \frac{\lambda}{(\lambda+\gamma) - \kappa e^{-\psi(t)}} \Rightarrow m^* = \frac{\lambda}{\lambda+\gamma} \quad (13)$$

where: $\kappa = 1 - \frac{m^*}{m(0)}$, $\psi = \frac{(\lambda+\gamma)^2}{\lambda}$.

- Probability density of fluctuations:

$$p(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \quad : \quad \sigma^2 = m^* \frac{\gamma}{\lambda+\gamma} \quad (14)$$

Asymptotic solutions: stationary equilibrium

- Trend dynamics and stationary state:

$$m(t) = \frac{\lambda}{(\lambda+\gamma)-\kappa e^{-\psi(t)}} \Rightarrow m^* = \frac{\lambda}{\lambda+\gamma} \tag{13}$$

where: $\kappa = 1 - \frac{m^*}{m(0)}$, $\psi = \frac{(\lambda+\gamma)^2}{\lambda}$.

- Probability density of fluctuations:

$$p(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \quad : \quad \sigma^2 = m^* \frac{\gamma}{\lambda+\gamma} \tag{14}$$

Asymptotic solutions: stationary equilibrium

- Trend dynamics and stationary state:

$$\boxed{m(t) = \frac{\lambda}{(\lambda+\gamma)-\kappa e^{-\psi(t)}} \;\Rightarrow\; m^* = \frac{\lambda}{\lambda+\gamma}} \tag{13}$$

where: $\kappa = 1 - \frac{m^*}{m(0)}$, $\psi = \frac{(\lambda+\gamma)^2}{\lambda}$.

- Probability density of fluctuations:

$$\boxed{p(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \;:\; \sigma^2 = m^* \frac{\gamma}{\lambda+\gamma}} \tag{14}$$

Endogenous formulation for η in asymptotic conditions I

- The dynamics of the economy is fully described by means of transition rates:

$$\lambda = \zeta\eta$$
$$\gamma = \iota(1-\eta)$$

- micro effects ζ and ι: probability and survival function of $\tilde{u}$;
- macro effect η: identification of its equilibrium formulation.

Endogenous formulation for η in asymptotic conditions I

- The dynamics of the economy is fully described by means of transition rates:

$$\lambda = \zeta\eta$$
$$\gamma = \iota(1 - \eta)$$

- micro effects ζ and ι: probability and survival function of $\tilde{u}$;
- macro effect η: identification of its equilibrium formulation.

Endogenous formulation for η in asymptotic conditions I

- The dynamics of the economy is fully described by means of transition rates:

$$\lambda = \zeta\eta$$
$$\gamma = \iota(1 - \eta)$$

- micro effects ζ and ι: probability and survival function of $\tilde{u}$;
- macro effect η: identification of its equilibrium formulation.

Endogenous formulation for η in asymptotic conditions I

- The dynamics of the economy is fully described by means of transition rates:

$$\lambda = \zeta\eta$$
$$\gamma = \iota(1-\eta)$$

- micro effects ζ and ι: probability and survival function of $\tilde{u}$;
- macro effect η: identification of its equilibrium formulation.

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**: Markov random field $\leftrightarrow$ Gibbs random field.

$$\Downarrow$$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t) g(N^1)}}{N}} \tag{15}$$

where:

$$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t) - y^0(t)\right)^{-1}$$

$$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta} ln\left(\frac{N^1}{N-N^1}\right)$$

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**:
  Markov random field $\leftrightarrow$ Gibbs random field.

$\Downarrow$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t)g(N^1)}}{N}} \tag{15}$$

where:

$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t)-y^0(t)\right)^{-1}$

$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta}\ln\left(\frac{N^1}{N-N^1}\right)$

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**: Markov random field $\leftrightarrow$ Gibbs random field.

$$\Downarrow$$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t)g(N^1)}}{N}} \tag{15}$$

where:

$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t)-y^0(t)\right)^{-1}$

$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta}\ln\left(\frac{N^1}{N-N^1}\right)$

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**: Markov random field $\leftrightarrow$ Gibbs random field.

$$\Downarrow$$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t) g(N^1)}}{N}} \tag{15}$$

where:

$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t) - y^0(t)\right)^{-1}$

$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta} ln\left(\frac{N^1}{N-N^1}\right)$

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**: Markov random field $\leftrightarrow$ Gibbs random field.

$$\Downarrow$$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t) g(N^1)}}{N}} \tag{15}$$

where:

$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t)-y^0(t)\right)^{-1}$

$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta} ln\left(\frac{N^1}{N-N^1}\right)$

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**: Markov random field $\leftrightarrow$ Gibbs random field.

$$\Downarrow$$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t) g(N^1)}}{N}} \tag{15}$$

where:

$$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t) - y^0(t)\right)^{-1}$$

$$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta} ln\left(\frac{N^1}{N-N^1}\right)$$

Endogenous formulation for η in asymptotic conditions II

- **Detailed balance condition**: probability of influxes equals probability of outfluxes for all states $\Rightarrow$ master equation $= 0$.
- **Hammersley and Clifford theorem**: Markov random field $\leftrightarrow$ Gibbs random field.

$$\Downarrow$$

$$\boxed{\eta(N^1) = \frac{e^{\beta(t)g(N^1)}}{N}} \quad (15)$$

where:

$$\beta(t) = \ln\left(-\frac{y^1(t)-\bar{y}(t)}{y^0(t)-\bar{y}(t)}\right)\left(y^1(t) - y^0(t)\right)^{-1}$$

$$g(N^1) = -\frac{1}{2\beta}\frac{dH(N^1)}{dN^1} = -\frac{1}{2\beta}\ln\left(\frac{N^1}{N-N^1}\right)$$

System's dynamics

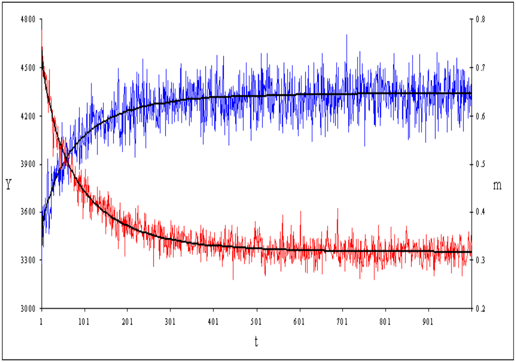

Figure: Trends (black lines) and series for *m* (red line, right axis) and value of aggregate production (blue line).

Effects of r

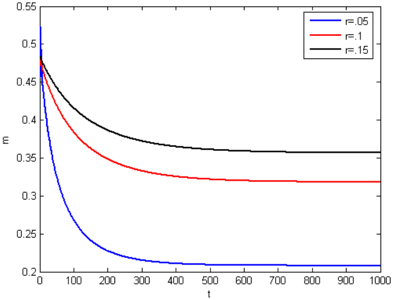

Figure: Different levels of m^* for different interest rates.

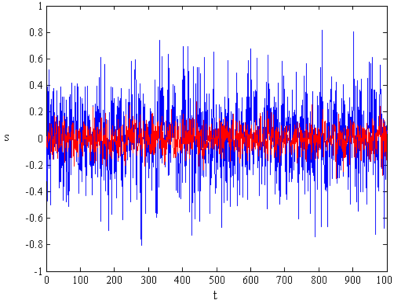

Figure: Spread for $r = 0.1$ (blue) and $r = 0.05$ (red).

Bifurcation diagram

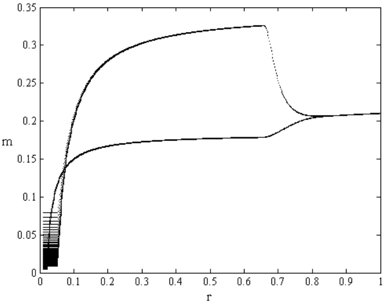

Figure: Bifurcation diagram for m as a function of the interest rate r.

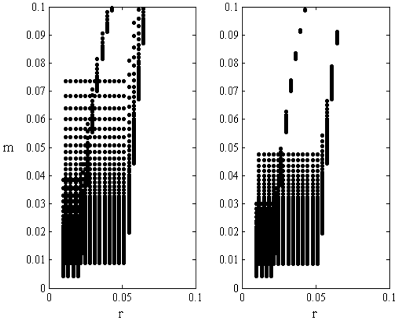

Figure: Particular of bifurcation diagrams with $m(0) = 0.4$ (left) and $m(0) = 0.1$ (right).

Non listed firms: Italy and France

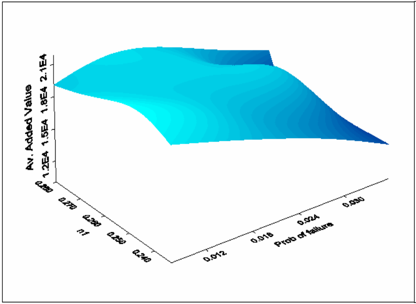

Figure: Italy 1992-2005. Average added value as a function of μ and n^1.

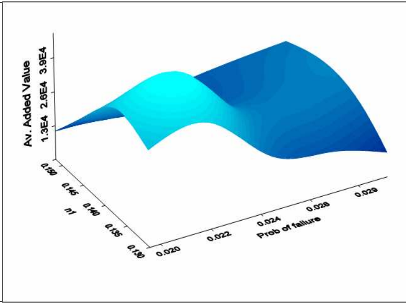

Figure: France 1992-2005. Average added value as a function of μ and n^1.

Listed firms: USA

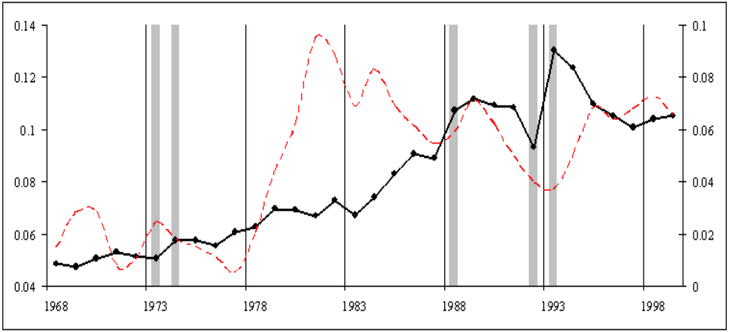

Figure: Trends of fraction of US listed firms with equity ratio below 0.1 (black line) and real lending interest rate (red dashed line, right axis). Grey areas: rejection of detailed balance.

References I

Aoki, Masanao and Yoshikawa, Hiroshi
Reconstructing Macroeconomics,
Cambridge University Press, 2006.

Delli Gatti, D. and Di Guilmi, C. and Gaffeo, E. and Giulioni, G. and Gallegati, M. and Palestrini, A.,
A new approach to business fluctuations: heterogeneous interacting agents, scaling laws and financial fragility,
Journal of Economic Behavior and Organization, 2005, 56(4).

Gallegati, Marco
Una generalizzazione dell'approccio Greenwald-Stiglitz con imprese eterogenee,
Economia Politica, 2002

References II

Greenwald, Bruce and Stiglitz, Joseph,
Financial markets imperfections and business cycles,
Quarterly journal of Economics, 1993, num.108

Minsky, Hyman
Inflation, recession and economic policy, New York: ME Sharpe, 1982.

Myers, Stewart C. and Majluf, Nicholas S.
Corporate Financing and Investment Decisions when Firms Have Information that Investors do not have,
Journal of Financial Economics, 1984.