Cycles, Growth, and the Great Recession

Economic Reflections in Times of Uncertainty

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7 A Kaleckian macromodel with memory

Gian Italo Bischi and Ahmad Naimzada

In a recent paper, Charles (2010) considers a post-Keynesian model, based on Delli Gatti, Gallegati, and Gardini (1993), and proposes a particular form of endogenous propensity to invest, denoted as abnormal, such that a fall in the propensity to invest occurs when capacity utilization (measured as the output–capital ratio) increases. As argued by Charles, this is attributed to strong pressures from shareholders represented by institutions like banks or pension funds in a financial capitalist economy. In fact, such stockholders may refuse the investment policy needed to respond to a rise in output because it threatens the profitability of their assets in the short run, as the decision to invest may involve a prolonged rise in the retention rate, to ensure safe growth through larger internal funds, leading to a decrease in the amount of their dividends. The dynamic model obtained by Charles is expressed by a one-dimensional quadratic map (conjugate to the standard logistic map) that may exhibit persistent cycles and chaotic trajectories as a result of the abnormal behavior.

In this chapter we assume that, instead of being a function of only the current capacity utilization, the (abnormal) endogenous propensity to invest also depends on the previous value through a weighted average: that is, the stockholders are assumed to have a certain degree of memory, which may be interpreted as more prudent behavior. This assumption gives rise to a two-dimensional discrete dynamical system that includes a behavioral parameter that represents the degree of memory (or level of prudence) of the stockholders. The qualitative analysis of this dynamical system shows that memory has a stabilizing role for a weighted average close to a uniform distribution; otherwise, when the weighted average (i.e. the memory parameter) takes extreme values (i.e. consideration of current value prevails or it is too much neglected), the system is destabilized through period-doubling or Neimark–Sacker bifurcations respectively. Through a stability analysis performed by the usual linearization procedure, we analytically prove the occurrence of the local bifurcations that cause stability loss of the unique equilibrium, and the delimitation of the stability region in the space of parameters is obtained. However, a numerical investigation of the global dynamic behaviors of the model clearly shows that further dynamic scenarios must be considered, characterized by coexistence of attracting sets, each with its own basin of attraction, even in the range of stability of the unique steady state. The possibility of coexistence of the locally stable steady state with different cyclic or chaotic attractors that characterize different kinds of long-run behavior, is shown numerically, as well as the structure of the basins of attraction that characterize the path dependence of the model with memory. This is an interesting situation because it gives an intermediate alternative between two antagonistic points of view, one stating stability of the equilibrium (i.e. any perturbation from the stationary equilibrium is recovered by the endogenous dynamics that goes back to the equilibrium in the long run) and the other one stating instability (i.e. any small perturbation from the equilibrium is amplified by the endogenous dynamics, thus leading to a different attractor characterized by endless self-sustained oscillations). In the case of coexistence, a situation is obtained, sometimes denoted by the term "corridor stability" after Leijonhufvud (1973), such that small perturbations are recovered as far as they are confined inside the basin of attraction of the locally stable equilibrium, whereas larger perturbations lead to time evolutions that further depart from the equilibrium and go to the coexisting attractor in the long run, where oscillatory motion prevails (periodic or chaotic). Moreover, the situation may be even more involved when the boundaries that separate the two basins assume a complicated shape (sometimes quite convoluted), as we shall show numerically in the model considered in this chapter.

These dynamic scenarios, together with their economic consequences, clearly show the importance of a global analysis of nonlinear dynamical systems, which can often be performed only through heuristic methods obtained by a combination of analytical, geometrical, and numerical methods. In fact, a study limited to local stability and bifurcations, based on the linear approximation of the model around the equilibrium points, sometimes may be quite incomplete and even misleading, as the example considered in this chapter clearly shows.

The plan of the work is as follows. In the next section we describe the model with memory and we prove analytical results about the role of memory parameter on the local stability of its unique steady state. We then confirm the stability results numerically and we investigate some global properties of the model and the different kinds of disequilibrium dynamics observed. Moreover, the possibility of coexistence of locally stable steady state and different kinds of cyclic attractor is shown numerically, as well as the structure of the basins of attraction that characterize the path dependence of the model with memory.

The model and its local stability properties

We consider a closed economy without government intervention. National income is subdivided into wages and profits:

$$pY = wL + \Pi \tag{1}$$

where $p \geq 0$ is the price level, $Y \geq 0$ is the national output, $w \geq 0$ is the nominal wage rate, $L \geq 0$ is the level of employment and $\Pi \geq 0$ represents the level of gross profits. Firms, assumed to be all identical, set the price by the following markup-pricing equation:

$$p = (1+m)wl \tag{2}$$

where $m \geq 0$ is the fixed markup used by firms and $l \geq 0$ is the fixed labor–output ratio $l = \frac{L}{Y}$. From equations (1) and (2) we get the profit share in terms of nominal income $0 \leq \pi \leq 1$:

$$\pi = \frac{\Pi}{pY} = 1 - \frac{wl}{p} = \frac{m}{1+m} \tag{3}$$

For the sake of simplicity, in the following we assume $p = 1$. The rate of profit r, defined as the fraction of profits in nominal capital stock, can be expressed as:

$$r = \frac{\Pi}{K} = \frac{Y}{K}\frac{\Pi}{Y} = u\pi \tag{4}$$

where K is the capital equipment and u, the output–capital ratio, is a proxy for the rate of capacity use.

We assume that the economy is populated by three kinds of agent: firms, capitalists, and workers. Turning our attention to saving behaviors, we postulate that firms save a portion $0 \leq s_f \leq 1$ of their net profits $(r - id)$, and capitalists save a portion $0 \leq s_c \leq 1$ of their revenues, including distributed dividends $(1 - s_f)(r - id)$ and interest received from firms, id.

Workers are assumed to consume all their income. Thus the global saving function assumes differentiated propensity to save:

$$g^s = \frac{S}{K} = s_f(r - id) + s_c\left[(1 - s_f)(r - id) + id\right] \tag{5}$$

where S is the total saving. Investment demand, as a ratio of capital stock, is given by:

$$g^d = \frac{I}{K} = \alpha + \beta s_f (r - id) \tag{6}$$

where $\alpha \geq 0$ represents animal spirits and $\beta \geq 0$ represents the propensity to invest. Assuming the standard Keynesian adjustment mechanism – that is, output, through the rate of capacity utilization, changes according to the excess of demand in the goods market – we get the dynamic equation:

$$u_{t+1} = u_t + g_t^d - g_t^s$$

where u_t is the output–capital ratio, used as a proxy for the rate of capacity use, at time period t, and πu_t represents the macroeconomic profit rate at time t. So, after some substitutions and straightforward algebraic manipulations, the same difference equation proposed by Charles (2010) is obtained:

$$u_{t+1} = \left[\alpha - s_f(\beta_t + s_c - 1)id\right] + \left\{1 - \pi\left[s_f(1 - \beta_t) + s_c(1 - s_f)\right]\right\} u_t \tag{7}$$

In Charles (2010) the propensity to invest is endogenized by assuming:

$$\beta_t = \beta_1 - \beta_2 u_t \tag{8}$$

with $\beta_i > 0$, i = 1,2, in order to indicate that a rise in the rate of capacity utilization causes a fall in the propensity to invest. As argued in Charles (2010), this apparent paradox is explained by incorporating the presence of stockholders, represented by institutions like banks and pension funds, which may refuse the investment policy needed to respond to a rise in output because it threatens the profitability of their assets in the short run. For example, the decision to invest may perfectly involve an immediate and prolonged rise in the retention rate, to ensure safe growth through larger internal funds. This is unacceptable for shareholders with short-term views since it would mechanically diminish the amount of their dividends. In a financial capitalist economy, the primacy of stockholders is such that they have the capability to ask for abnormal requests, and postponing accumulation projects is obviously one of them. Therefore, the existence of omnipotent shareholders explains why an increase in the rate of capacity utilization may lead to a fall in the propensity to invest. Such a situation is denoted by Charles as an "abnormal case."

By reducing the first-order difference equation obtained from (7) with (8) to a quadratic map (conjugate to the standard logistic map), Charles (2010) shows that when firms' managers adopt such abnormal behaviors, the system may exhibit persistent cycles and chaotic trajectories as a result of the increasing pressures from shareholders regarding the propensity to invest. In fact, (7) with (8) becomes a one-dimensional quadratic map, conjugate to the logistic map (see, for example, Devaney, 1987; Lorenz, 1993) well known for its chaotic dynamics.

In this chapter we assume that, instead of being a function of only the current capacity utilization of the economic system, u_t, the endogenous propensity to invest also depends on the previous value u_{t-1}, i.e. the stakeholders have a certain degree of memory. This assumption can be expressed by using the following weighted average to compute the propensity to invest.

$$\beta_t = \beta_1 - \beta_2 \left[(1-\omega) u_t + \omega u_{t-1} \right] \tag{9}$$

where the real parameter $\omega \in [0, 1]$ represents a memory parameter, as the expression (9) reduces to (8) in the limiting case $\omega = 0$, whereas in the other limiting case $\omega = 1$, the actual value u_t is neglected and only the previous one is considered in the determination of the propensity to invest. Of course, intermediate values of ω represent different kinds of weighted average between the two last observations of u_t, $\omega = 1/2$ corresponding to the case of uniform average.

If we plug (9) into (7) we obtain a second-order difference equation, as u_{t+1} is now influenced by both u_t and u_{t-1}, which can be written as an equivalent two-dimensional discrete dynamical system. In fact, by introducing the new dynamic variables $x_t = u_t$ and $y_t = u_{t-1}$ the model (2) with (9) assumes the form:

$$\begin{cases} x_{t+1} = -\pi B(1-\omega)x_t^2 - \pi B\omega x_t y_t + \left[A + idB(1-\omega)\right]x_t + idB\omega y_t + C \\ y_{t+1} = x_t \end{cases} \quad (10)$$

where the following aggregate parameters have been introduced:

$$A = 1 - \pi\left[s_f(1-\beta_1) + s_c(1-s_f)\right]$$
$$B = s_f\beta_2$$
$$C = \alpha - ids_f(\beta_1 + s_c - 1)$$

By imposing the steady-state condition $u_{t+1} = u_t$ for each t, corresponding to $x_{t+1} = x_t = y_t$, it is straightforward to see that a unique positive equilibrium exists, which does not depend on the memory parameter ω and is the same as in the model proposed by Charles:

$$x^* = u^* = \frac{A + idB - 1 + \sqrt{(A + idB - 1)^2 + 4\pi BC}}{2\pi B} \quad (11)$$

whose stability properties are strongly influenced by the memory parameter ω, as stated by the following proposition.

Proposition 1. *The equilibrium $E = (x^*, x^*)$ of the dynamical system (10) is locally asymptotically stable if $\omega_f < \omega < \omega_h$ with*

$$\omega_f = \frac{\sqrt{(A + idB - 1)^2 + 4\pi BC} - 2}{A - idB - 1 + \sqrt{(A + idB - 1)^2 + 4\pi BC}} \quad (12)$$

and

$$\omega_h = \frac{2}{A - idB - 1 + \sqrt{(A + idB - 1)^2 + 4\pi BC}} \quad (13)$$

If the memory parameter ω exits the stability interval decreasing through the lower bound ω_f then it loses stability through a supercritical flip bifurcation, at which a stable cycle of period two is created, whereas if ω exits the stability interval increasing through the upper bound ω_h then it loses stability through a supercritical Neimark–Sacker bifurcation at which a stable closed invariant curve is created surrounding E, along which quasi periodic or periodic motion occurs.

Proof. The Jacobian matrix of (10) computed at the equilibrium *E* becomes:

$$J(E)=\begin{bmatrix} \pi B(\omega-2)x^* + A + idB(1-\omega) & -\pi B\omega x^* + idB\omega \\ 1 & 0 \end{bmatrix} \tag{14}$$

Let $Tr = \pi B(\omega-2)x^* + A + idB(1-\omega)$ and $Det = \pi B\omega x^* - idB\omega$ be, respectively, the trace and the determinant of the matrix *J(E)*. Then the characteristic equation becomes:

$$P(z) = z^2 - Tr\cdot z + Det = 0$$

and a set of sufficient conditions for the stability of *E*, i.e. for the eigenvalues to be inside the unit circle of the complex plane, is given by:

$$P(1) = 1 - Tr + Det > 0; \quad P(-1) = 1 + Tr + Det > 0; \quad 1 - Det > 0 \tag{15}$$

In our case we have $P(1) = 1 + 2\pi Bx^* - A - idB = \sqrt{(A + idB - 1)^2 + 4\pi BC} > 0$ for each set of parameters, where the expression (11) has been used. Instead, the other two stability conditions become, respectively:

$$\omega > \frac{2\pi Bx^* - A - 1 - idB}{2B(\pi x^* - id)} = \omega_f \quad \text{and} \quad \omega < \frac{1}{B(\pi x^* - id)} = \omega_h$$

where the expressions of the bifurcation values ω_f and ω_h given in the proposition are obtained by inserting (11) inside these expressions. The value of ω at which P (− 1) ecomes negative represents a flip (or period-doubling) bifurcation value at which an eigenvalue exits the unit circle through the value −1, and the one at which 1−*Det* becomes negative represents a Neimark–Sacker bifurcation at which a couple of complex and conjugate eigenvalues exit the unit circle of the complex plane (see, for example, Guckenheimer and Holmes, 1983; Lorenz, 1993). We claim numerical evidence for the supercritical nature of these bifurcations, as shown in the next section.

In Figure 7.1 the stability region bounded by the two curves $\omega = \omega_h$ (flip bifurcation curve) and $\omega = \omega_h$ (Neimark–Sacker bifurcation curve) is represented by the grey-shaded region in the parameters' plane (β_2, ω) for the same set of parameters used in Charles (2010), namely $\alpha = 0.2,\ i = 0.05,\ d = 0.3,\ \pi = 0.47,\ s_f = 0.8,\ s_c = 0.9,\ \beta_1 = 5.5.$

As can be seen, along the line $\omega = 0$, at $\beta_2 = 5.5$ the equilibrium loses stability for increasing values of β_2 through a flip bifurcation, followed by the well-known period-doubling route to chaos for further increasing values of β_2. For higher values of the memory parameter ω the range of stability of β_2 is increased and it becomes larger and larger for intermediate values of ω: that is, for a more uniform memory distribution. However, this is no longer true for high values of

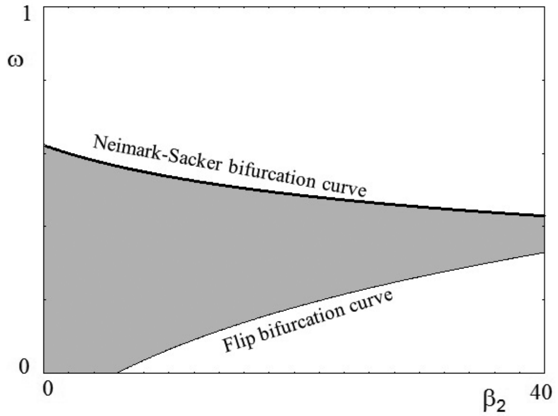

Figure 7.1 Stability region of the equilibrium E, represented by the grey-shaded region in the parameters' plane (β_2, ω) for the same set of parameters used in Charles (2010): $\alpha = 0.2$, $i = 0.05$, $d = 0.3$, $\pi = 0.47$, $s_f = 0.8$, $s_c = 0.9$, $\beta_1 = 5.5$.

Source: Authors' Own

ω: that is, when in (9) more weight is given to the previous value u_{t-1}. In this case a new phenomenon occurs, strictly related with the memory effect: for increasing values of β_2 the stability of the equilibrium is lost through a Neimark–Sacker bifurcation – quasi-periodic motion around E is observed. Moreover, the range of stability of β_2 is reduced for increasing values of ω. It is interesting to observe the qualitative changes in the long-run dynamics when the memory parameter ω is increased for a fixed value of β_2. This is shown in the bifurcation diagram of Figure 7.2, obtained for $\beta_2 = 17$ and increasing values of the bifurcation parameter ω. This bifurcation diagram clearly shows the stability range of the equilibrium E for intermediate values of the memory parameter ω, as well as the two different kinds of local bifurcation through which E loses stability. This bifurcation diagram confirms the results given in the proposition on local stability and bifurcations of the unique equilibrium E.

For increasing values of ω in the region $\omega > \omega_h$ the amplitude of the quasi-periodic motion increases until the closed invariant curve changes its shape and becomes a chaotic attractors which suddenly disappears, and the generic trajectory diverges. This is caused by global (or contact) bifurcations, as will be shown numerically in the next section. However, other interesting dynamic phenomena will be shown by a global numerical exploration that cannot be revealed by the local stability given in this section.

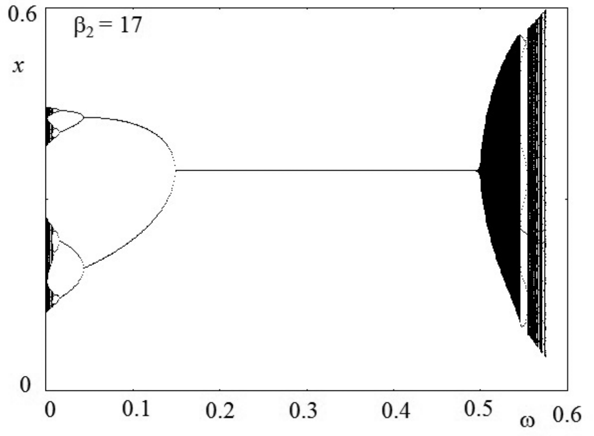

Figure 7.2 Bifurcation diagram, with bifurcation parameter ω, obtained with $\beta_2 = 17$ and all the other parameters at the same values as in Figure 7.1. For each value of the bifurcation parameter ω a trajectory of the model (10) is generated, starting f[illegible]e initial condition $(x_0, y_0) = (0.2, 0.2)$.

Source: Authors' Own

Disequilibrium dynamics, multistability, and basins of attraction

In this section we perform some numerical explorations in order to give a more complete view of the global dynamic properties of the model. In fact, the local stability analysis of the previous section, based on the usual linearization procedure that only gives information about the model's behavior in a neighborhood of the equilibrium point, gives no insight into the size and the shape of the basin of attraction of the stable equilibrium: that is, the robustness of its stability with respect to exogenous perturbations. Moreover, for sets of parameters such that the equilibrium point is not stable, we need to analyze the kind of disequilibrium dynamics that prevails, and even when the equilibrium is locally stable we have to check if other coexisting attractors are present far from the equilibrium point: in other words, if other kinds of feasible long-run dynamics can be observed, starting from initial conditions outside of the basin of *E*.

First of all, Figure 7.3 shows the representations, in the phase space *(x,y)*, of the attractors and the basins of attraction for the same set of parameters used to obtain the bifurcation diagram of Figure 7.2 and three different values of the memory parameter ω corresponding to the three different dynamic scenarios stressed in the proposition and shown in the bifurcation diagram: Figure 7.3a, obtained with $\omega = 0$ (no-memory case), shows a the two-cyclic chaotic attractor whose basin of attraction is represented by the white region, the black one being the basin of infinity – that is, the set of initial conditions that generate diverging

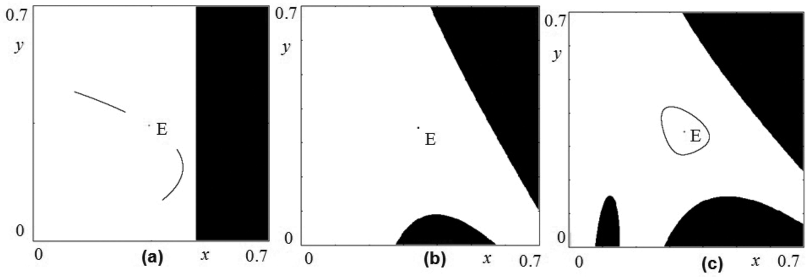

Figure 7.3 Three representations of the phase plane of the model (map) obtained with the same parameters as in Figure 7.2 with $\omega = 0 < \omega_f \approx 0.14$ in panel (a); $\omega = 0.4 \in (\omega_f, \omega_h) \approx (0.14, 0.5)$ the stability range, in panel (b); $\omega = 0.51 > \omega_h \approx 0.5$ in panel (c). The black region represents the set of diverging trajectories and the white region represents the basin of attraction of the attractor shown: the two-cyclic chaotic set in panel (a), the equilibrium E in panel (b), a stable closed inv[illegible] orbit along which quasi-periodic motion occurs in panel (c).

Source: Authors' Own

trajectories; in Figure 7.3b, obtained for $\omega = 0.4$, the equilibrium point E is the only feasible (i.e. positive and bounded) attractor; in Figure 7.3c, obtained for $\omega = 0.51$ – that is, just after the Neimark–Sacker bifurcation (occurring at the value $\omega_h \approx 0.5$ computed according to the proposition) – the stable closed invariant curve is shown, surrounding the unstable equilibrium E, along which the dynamic variable x_t exhibits a quasi-periodic motion.

As the memory parameter ω increases, the closed invariant curve enlarges: that is, the amplitude of the oscillations increases, until it reaches the boundary of the basin of attraction, as shown in Figure 7.4, obtained for $\omega = 0.588$. After the contact between the attractor and the boundary of its basin, the attractor disappears, through a global bifurcation known as final bifurcation (see Mira et al., 1996) or boundary crisis (Grebogi, Ott, and Yorke, 1983). After this global bifurcation the generic trajectory is divergent. This is the reason why the bifurcation diagram shown in Figure 7.2 is interrupted before the bifurcation parameter reaches the value 0.7. Of course, this information cannot be obtained from an analytical study of the linear approximation of the map.

Other surprising dynamic phenomena caused by the presence of the memory can be seen for slightly higher values of the parameter β_2. In fact, the bifurcation diagram obtained for $\beta_2 = 20.5$, shown in Figure 7.5, exhibits the coexistence of two different attractors for intermediate values of the memory parameter ω. In fact, such a bifurcation diagram is obtained by taking, for each value of the bifurcation parameter ω, two different initial conditions $(x_0, y_0) = (0.3, 0.3)$ and $(x_0, y_0) = (0.02, 0.02)$ respectively, and the asymptotic portions of the corresponding trajectories are represented. As can be seen in the diagram, in the range $\omega \in (0.17, 0.48)$, i.e. for intermediate values of the memory parameter such that the

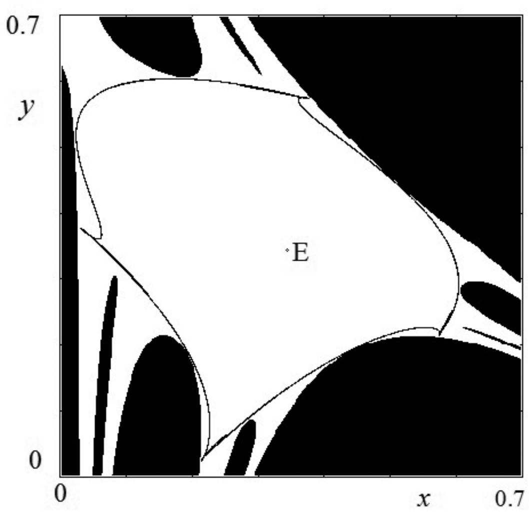

Figure 7.4 The attractor around E for $\omega = 0.588$ (which is a chaotic attractor at this stage) is very close to the boundary of its basin, and will disappear through a final (or boundary crisis) global bifurcation after a slight increase of the parameter

Source: Authors' Own

equilibrium E is locally asymptotically stable according to the proposition of the previous section, a different kind of asymptotic behavior can be obtained, given by oscillatory (periodic or chaotic) motion, represented by an alternative attractor with its own basin that shares the phase plane with the basin of the locally stable equilibrium. A representation of the two coexisting attractors, as well as their basins, obtained with $\omega = 0.3$, is shown Figure 7.6, where the basin of E is represented by the white region, the basin of the coexisting bounded attractor (a cyclic chaotic attractor at this stage) by the grey region, and the basin of diverging trajectories by the black region, as usual. It is worth noticing that such a coexistence could not be predicted by any analytical local analysis of the dynamical system, and if the analysis is limited to the proof of the proposition of the previous section, together with its immediate numerical confirmation given by the bifurcation diagram of Figure 7.2 or the numerical simulations of Figure 7.3, then a quite incomplete, and even misleading, description of the dynamic properties of the model considered would be given, stating that the system shows convergence to the unique equilibrium for intermediate values of the memory parameter ω. Instead, after our numerical explorations we can state that for a wide range of the parameters of the model (such as the one giving the abnormal propensity to

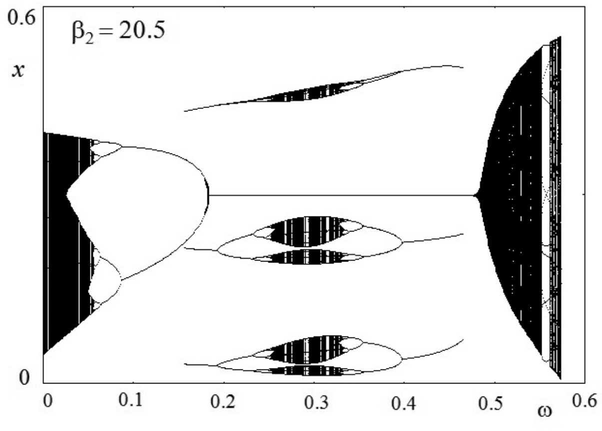

Figure 7.5 Bifurcation diagram, with bifurcation parameter ω, obtained with $\beta_2 = 20.5$ and all the other parameters at the same values as in Figure 7.2. For each value of the bifurcation parameter ω two trajectories of the model (map) are generated, starting from the initial conditions $(x_0, y_0) = (0.3, 0.3)$ and $(x_0, y_0) = (0.02, 0.02)$ respectively. In the range $\omega \in (0.17, 0.48)$ the two trajectories converge to different attractors, the equilibrium point E and an oscillatory (periodic or chaotic) one respectively.

Source: Authors' Own

invest β_2) the local stability of the equilibrium E guarantees that the system will converge to it only if the initial conditions are taken sufficiently close to the equilibrium, and larger perturbations will lead the system to exhibit self-sustained bounded oscillations (periodic or chaotic) in the long run.

This situation has been extensively discussed in the economic literature and has sometimes been called "corridor stability" (see, for example, Leijonhufvud, 1973; Dohtani, Inaba, and Osaka, 2007). This stream of literature stresses the fact that nonlinear dynamic models may have the property that small perturbations are recovered as far as they are confined inside the basin of attraction of a locally stable equilibrium, whereas larger perturbations lead to time evolutions that depart further from the equilibrium and go to the coexisting attractor in the long run. In other words, a neoclassical view of the economic system, spontaneously recovering its equilibrium configuration, prevails if small perturbations are considered, whereas a neo-Keynesian view of amplifications of the perturbations, leading to out-of-equilibrium asymptotic dynamics, prevails when larger perturbations are considered. However, the situation may be even more involved when the boundaries that separate the two basins assume a complicated shape (sometimes quite convoluted). For example, if we consider three different initial conditions with

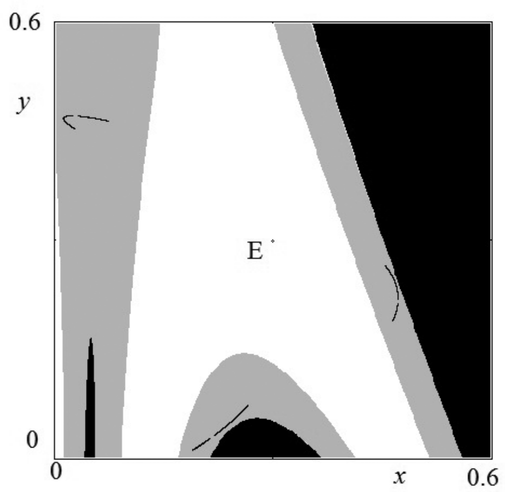

Figure 7.6 Basins of attractions with $\omega = 0.3$ and all the other parameters as in Figure 7.5. The basins of attraction of the coexisting attractors are represented by different colors: white for the basin of E, grey for the basin of the cyclic chaotic attractor, black for the basin of diverging trajectories.

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the set of parameters used to obtain the dynamic scenario of Figure 7.6, we obtain the usual convergence of a trajectory generated by an initial condition close to the equilibrium, like the one shown in panel (a) of Figure 7.7, and convergence to the chaotic attractor starting from an initial condition taken at a greater distance, like the one shown in panel (b), as well as a counterintuitive situation of convergence starting from an even further initial condition, as in the case shown in panel (c). Of course, all the three asymptotic situations shown in Figure 7.7 can be forecasted on the basis of the basins' representation of Figure 7.7. It is plain that, again, all these typically nonlinear phenomena could not be forecasted on the basis of the analytical results of the previous section. So, such dynamic scenarios, together with their economic consequences, clearly show the importance of a global analysis of nonlinear dynamical systems, which can often be performed only through an heuristic method obtained by a combination of analytical, geometrical, and numerical methods. It is also worth stressing that this path dependence cannot be observed in the model without memory proposed by Charles (2010), corresponding to our limiting case $\omega = 0$, because in that case the system dynamics are governed by a one-dimensional quadratic map – a typical map with negative Schwartzian derivative and just one critical point, and hence a unique attractor (see, for example, Devaney, 1987).

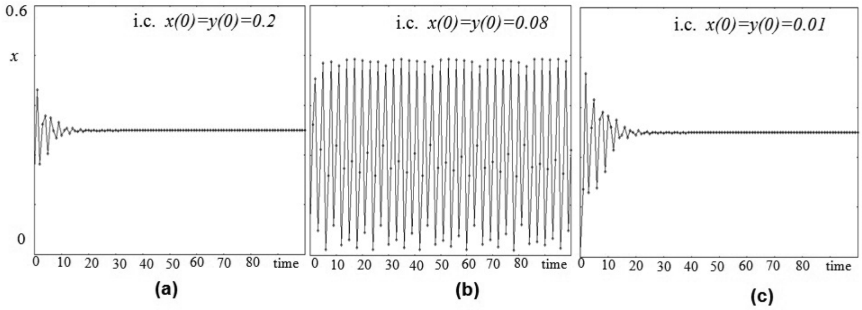

Figure 7.7 Three trajectories of the model versus time, for the same set of parameters as in Figure 7.6, from three different initial conditions: $(x_0, y_0) = (0.2, 0.2)$ in panel (a), $(x_0, y_0) = (0.08, 0.08)$ in panel (b), $(x_0, y_0) = (0.01, 0.01)$ in panel (c).

Source: Authors' Own

Conclusion

We have proposed a modification of the model proposed by Charles (2010) by introducing a memory parameter into the abnormal (that is, negatively sloped) relation between the propensity to invest and the rate of capacity utilization: in other words, instead of a function of only the capacity utilization of the previous period, the propensity to invest has been related to a weighted average of the capacity utilization observed in the last two periods, which transforms the one-dimensional model of Charles into a two-dimensional one. The introduction of memory effects, represented by convex combinations of previous states, has been considered by many authors as a realistic assumption in some economic systems (see, for example, Invernizzi and Medio, 1991). As shown in Aicardi and Invernizzi (1992), sometimes an increasing memory – that is, a larger weight given to the past realizations – has a stabilizing effect. This idea is partially confirmed in the model studied in this chapter, in the sense that starting from a situation of chaotic dynamics with zero memory, as given in the paper by Charles, a situation of stability is reached through a sequence of backward flip (or period-halving) bifurcations. However, we have also proved that the equilibrium loses stability again for further increases in value of the memory parameters through a Neimark–Sacker bifurcation. Indeed, the proposition proved in the second section above essentially states that the unique equilibrium of the model is stable for intermediate values of the memory parameter: that is, for a memory distribution close to a uniform average of the two past values of capacity utilization. This result is quite intuitive, and is also observed in the presence of longer memories, such as the fading memory involving all the states observed in the past, as in the model considered in Bischi and Naimzada (1997). However, as stressed by Hommes et al. (2012), the role of memory and time horizons has hardly been studied in the literature, and its role in a general dynamic framework is not a simple matter. Indeed, even in the case of a short memory, as we considered in the model analyzed in this chapter, things become quite complex, as in some ranges on the parameters such that the equilibrium is locally stable, coexisting periodic and chaotic attractors have been observed numerically, thus giving a strong path dependence. In fact, when the locally stable equilibrium coexists with a different kind of attractor, periodic or chaotic, each with its own basin of attraction, a typical situation of "corridor stability" occurs, as small perturbations (or shocks or historical accidents) around the equilibrium are endogenously recovered by the endogenous dynamics of the system, whereas larger perturbations are amplified by the endogenous dynamics, thus leading to a completely different (and non-stationary) disequilibrium dynamics, so that only an external control policy can force the system back to the original equilibrium. The situation becomes even more involved when the boundaries that separate the two basins assume a complicated (sometimes quite convoluted) shape, as we have shown numerically.

These dynamic scenarios clearly show the importance of a global analysis of nonlinear dynamical systems, which can often be performed only through a heuristic method obtained by a combination of analytical, geometrical, and numerical methods. In fact, an analytical study of the local stability and bifurcations, based on the linear approximation of the model around the equilibrium points, may sometimes be quite incomplete and even misleading.

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