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| Copyright Holder | Springer International Publishing Switzerland | |
| Author | Family Name | Bischi |
| | Particle | |
| | Given Name | Gian Italo |
| | Suffix | |
| | Email | gian.bischi@uniurb.it |
| Author | Family Name | Lamantia |
| | Particle | |
| | Given Name | Fabio |
| | Suffix | |
| | Email | lamantia@unical.it |
| Author | Family Name | Viganò |
| | Particle | |
| | Given Name | Elena |
| | Suffix | |
| | Email | elena.vigano@uniurb.it |
| Abstract | <p>In this paper we provide an overview of some recent dynamic models of commercial fisheries. Our starting points are the papers by Okuguchi (Keio Econ Stud 35:9–17, 1998) and Okuguchi and Szidarovszky Szidarovszky F, Okuguchi (Seoul J Econ 11(3):321–330, 1998; Seoul J Econ 13:471–476, 2000) on oligopoly competition in international fisheries, which are based on the assumptions of homogeneous fish population, homogeneous space and continuous time. Following also insightful hints given in Okuguchi (Keio Econ Stud 21:37–44, 1984) and Erjaee and Okuguchi (Keio Econ Stud 41:61–71, 2006), we describe how some heterogeneities can be introduced in fishery models. The examples provided in this paper include bioeconomic models of fisheries with several species and with different time scales, ranging from continuous-time to discrete-time dynamic models and also addressing hybrid models, where some variables evolve in continuous time whereas other ones change in (event-driven) discrete time. Fishery models where the aquatic environment is subdivided into adjacent patches, characterized by different fishing policies including, as a particular case, Marine Protected Areas, are also described. Finally, we describe a fishery models with cooperators and defectors and punishment policies to encourage cooperation. All the models described in this overview include an evolutionary mechanism, based on imitative behavior, through which fishers are allowed to change their harvesting strategy over time.</p> | |

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Evolutionary Oligopoly Models of Commercial Fishing with Heterogeneities

Gian Italo Bischi, Fabio Lamantia and Elena Viganò

Abstract In this paper we provide an overview of some recent dynamic models of commercial fisheries. Our starting points are the papers by Okuguchi (Keio Econ Stud 35:9–17, 1998) and Okuguchi and Szidarovszky Szidarovszky F, Okuguchi (Seoul J Econ 11(3):321–330, 1998; Seoul J Econ 13:471–476, 2000) on oligopoly competition in international fisheries, which are based on the assumptions of homogeneous fish population, homogeneous space and continuous time. Following also insightful hints given in Okuguchi (Keio Econ Stud 21:37–44, 1984) and Erjaee and Okuguchi (Keio Econ Stud 41:61–71, 2006), we describe how some heterogeneities can be introduced in fishery models. The examples provided in this paper include bioeconomic models of fisheries with several species and with different time scales, ranging from continuous-time to discrete-time dynamic models and also addressing hybrid models, where some variables evolve in continuous time whereas other ones change in (event-driven) discrete time. Fishery models where the aquatic environment is subdivided into adjacent patches, characterized by different fishing policies including, as a particular case, Marine Protected Areas, are also described. Finally, we describe a fishery models with cooperators and defectors and punishment policies to encourage cooperation. All the models described in this overview include an evolutionary mechanism, based on imitative behavior, through which fishers are allowed to change their harvesting strategy over time. Some of these models have been motivated by projects or management experiences performed in the Adriatic Sea by Italian fishing authorities.

1 Introduction

Starting from the seminal work by Smith (1969) on commercial fishing, modelled by a system of two ordinary differential equations that describe the time evolution of a fish species harvested by a time-evolving number of fishing vessels, Koji Okuguchi addressed several relevant extensions, briefly recalled below. In Okuguchi (1984)

G.I. Bischi (✉)

Department of Economics, Society, Politics, University of Urbino, Urbino, Italy
e-mail: gian.bischi@uniurb.it

he considered two interacting fish populations, preys and predators, both harvested and sold at fixed prices, with landing cost proportional to the square of harvest and inversely proportional to available fish stock. In 1998 Okuguchi studied a model of international trade of a single species, harvested by two countries under imperfect competition in an open-access sea; different prices are present in each country, obtained through two linear demand functions, which depend on the total amount of fish sold in each country by both local and foreign fishers. The questions of existence and stability of Nash equilibria for this duopoly model are studied under the assumption of a continuous-time logistic growth of the fish population. These results were then extended by Szidarovszky and Okuguchi in 1998 to the case of an oligopoly with $n \geq 2$ countries, and the effect of entry of a new fishing country on existence and stability of equilibria is also studied. The duopoly model of international imperfectly competitive fishery is re-considered by Erjaee and Okuguchi (2006) in a discrete-time setting, and sequences of bifurcations leading to periodic and chaotic oscillations are studied, both analytically and numerically.

Following this stream of literature, several dynamic bioeconomic models have been recently proposed, with the aim of describing new policies for sustainable exploitation of fisheries; the underlying assumptions of these works on fish growth, demand and cost functions are similar to the ones proposed in the above mentioned papers by Okuguchi and coauthors, so that the dynamic equations are still analytically tractable even if numerical methods have been used to confirm and extend the analytical results.

In this paper, we describe some of these fishery models, motivated by a research project, in which the authors have been involved, concerning the study of the effects of some harvesting policies and fishing limitations proposed by Italian authorities to regulate fisheries in the Adriatic Sea, see e.g. the book edited by Antonelli et al. (2005), as well as the papers Bischi et al. (2013a,b), dealing with a model with two fish species and a policy adopted in the Adriatic Sea that allows each fisher to fish only one with the possibility to switch to the other one at discrete-time periods, or Antonelli et al. (2006), Bischi and Lamantia (2007), Bischi et al. (2009), and Lamantia (2006), dealing with the creation of marine protected areas, or Bischi et al. (2004), dealing with different fishing strategies with the possibility to apply sanctions or taxes on the less environmental friendly fishing method.

The plan of the paper is as follows. Section 2 illustrates an evolutionary game for the exploitation of different species, where market and cost externalities help lowering harvesting pressure to the most endangered species. Section 3 describes an evolutionary model where exploiters can decide the aquatic region to harvest from, where different harvesting rules are established. Section 4 deals with a model where agents can decide whether to cooperate and harvest the amount that maximizes the utility of a coalition or to defect and maximize individual utility. Agents can switch over time between the two groups and can enforce cooperation through punishments. Finally, Sect. 5 concludes.

2 Two-Species Fisheries with Evolutionary Strategy Switching

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In Bischi et al. (2013a) a dynamic model is proposed to describe a fishery with two non-interacting species where a Fishing Regulatory Authority allows each fisher to harvest just one species during a given time period. Then fishers can reconsider their harvesting decision at discrete “switching times” on the basis of accrued profits. The model is motivated by an Italian fisheries management experience in the Northern Adriatic Sea, where this kind of “self-adjusting” fishing policy has been proposed to regulate harvesting of two shellfish species, with time periods of three years between switching times. Indeed, as the harvesting cost is inversely proportional to the available fish biomass, at each switching time more fishers are attracted towards the more abundant species, thus giving the possibility to restore the biomass of the scarcer resource.

To model this context, in Bischi et al. (2013a) a population of N fishers is considered, such that, at time t , $n_1(t)$ of them harvest only fish population 1, whose biomass density is $X_1(t)$, and $n_2(t)$ harvest only fish populations 2, whose biomass density is $X_2(t)$, with $n_1(t) + n_2(t) = N$. Let $r(t) = r_1(t) = n_1(t)/N$ be the fraction of fishers that at time t are allowed to harvest species 1. Of course, the complementary fraction $r_2(t) = 1 - r(t)$ represents the fraction of fishers that harvest species 2. The dynamics of the system is modelled assuming that natural growth and harvesting of the two species occurs in continuous time, whereas discrete-time strategy switching (decision-driven) regulates how agents adaptively update their harvesting strategy period by period, according to an evolutionary mechanism based on profit-driven replicator dynamics. Denoting by $G_i(X_i)$ the specific growth function of species i and by H_i its harvesting rate, the model assumes the following form:

$$\begin{cases} \dot{X}_1(t) = X_1(t)G_1(X_1(t)) - H_1(X_1(t), X_2(t)) \\ \dot{X}_2(t) = X_2(t)G_2(X_2(t)) - H_2(X_1(t), X_2(t)) \\ r(t) = \begin{cases} r(t-s) \frac{\bar{\pi}_1^*(t)}{r(t-s)\bar{\pi}_1^*(t) + [1-r(t-s)]\bar{\pi}_2^*(t)} & \text{if } \frac{t}{s} = \lfloor \frac{t}{s} \rfloor \\ r(\lfloor \frac{t}{s} \rfloor s) & \text{otherwise} \end{cases} \end{cases} \quad (1)$$

Here \dot{X}_i denotes the derivatives of biomass with respect to time, $\lfloor x \rfloor$ is the largest integer not greater than x (i.e. the floor of x), and

$$\bar{\pi}_i^*(t) = \frac{\int_{t-s}^t \pi_i^*(\tau) d\tau}{s} ; i = 1, 2 \quad (2)$$

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is the average profit observed by each agent at the end of every switching time of length s . So, the third equation in (1) states that at each switching time, each fisher, assumed to know the average profits of both groups during the previous period, compares the two average profits over the last fishing period. Average profit of each strategy is taken as fitness measure of that strategy, according to the paradigms of evolutionary game theory (see Weibull 1995; Hofbauer and Sigmund 1998): if $\bar{\pi}_1^*(t) > \bar{\pi}_2^*(t)$ then $r(t)$ increases, i.e. a fraction of fishers harvesting species 2 switch to harvest species 1, otherwise $r(t)$ decreases.

Following Szidarovszky and Okuguchi (1998), the harvesting rates $H_1(t)$ and $H_2(t)$ are computed according to the profit maximization problem of a Cournot oligopoly game. Assuming that the current total harvesting is wholly supplied to the market, prices are determined according to the following horizontal differentiated linear inverse demand system (see Singh and Vives 1984 for details)

$$\begin{cases} p_1 = f_1(H_1, H_2) = a_1 - b_1(H_1 + \sigma H_2) \\ p_2 = f_2(H_1, H_2) = a_2 - b_2(\sigma H_1 + H_2) \end{cases} \quad (3)$$

where a_i is the reservation price for species i , b_i represents the slope of the demand for fish i and $\sigma \in [0, 1]$ is the symmetric degree of substitutability between the two fish varieties. In particular, if $\sigma = 0$ then the two varieties are independent in demand. On the other hand, for $\sigma = 1$ they are perfect substitutes.¹ As usual in fishery oligopoly models, quadratic harvesting costs are assumed for both species, i.e. for harvesting h_i units of species i an agent faces a cost given by

$$C_i(X_i, h_i) = \gamma_i \frac{h_i^2}{X_i} \quad (4)$$

where γ_i measures the fishers' inefficiency for catching species i . This cost function is obtained and employed in Clark (1990), Okuguchi (1998), and Szidarovszky and Okuguchi (1998) and is very common in fishery models, since it captures the fact that it is easier and less expensive to catch fish if the fish population is large. Given these specifications of cost and price functions, the profit of fisher q belonging to group i ($i = 1, 2$) when harvesting $h_{i,q}$ reads

$$\pi_{i,q} = p_i h_{i,q} - \gamma_i \frac{h_{i,q}^2}{X_i}, \quad i = 1, 2 \quad (5)$$

¹In the context we are considering, we disregard the case $\sigma < 0$, related to varieties that are demand complementary.

Therefore, in deciding instantaneous harvesting of species i , the representative fisher q in group i solves the problem $\max_{h_{i,q}} \pi_{i,q}$. By taking the first order conditions and employing the symmetry property that players within each group are homogeneous (i.e. $h_{i,q} = h_{i,u}$, $i = 1, 2$;) we obtain for $i, j = 1, 2$ with $i \neq j$ the following harvesting quantities at a Nash equilibrium (see Bischi et al. 2013a for details) $h_i^*(X_i, X_j) =$

$$\frac{a_i X_i (b_j X_j [1 + N r_j] + 2 \gamma_j) - a_j b_i N r_j X_i X_j \sigma}{(b_i X_i (1 + N r) + 2 \gamma_i) (b_j X_j (1 + N (1 - r)) + 2 \gamma_j) - b_i b_j N^2 (1 - r) r X_i X_j \sigma^2}, \quad (6)$$

where $r_1 = r = \frac{n_1}{N}$ and $r_2 = (1 - r) = \frac{n_2}{N}$ represent, respectively, the fractions of fishers that harvest species 1 and 2, respectively. Hence, the harvesting terms in (1) become $H_1 = N r h_1^*$ and $H_2 = N (1 - r) h_2^*$ respectively. Moreover, by inserting (6) into (5), we get optimal individual profits

$$\pi_i^* = \left(b_i + \frac{\gamma_i}{X_i} \right) (h_i^*)^2 \quad (7)$$

which shows that profits are always non-negative.

Different kinds of growth functions can be considered in (1), according to the biological properties of the fish population and the marine environment at hand. In Bischi et al. (2013a), it is assumed that the two populations of (shell)fish follow a logistic natural growth rate of the form

$$G_i = \rho_i \left(1 - \frac{X_i}{k_i} \right); \quad i = 1, 2 \quad (8)$$

where ρ_i and k_i are, respectively, the intrinsic rate of growth and the carrying capacity of species i .

In order to investigate the model proposed by using both analytical and numerical methods, Bischi et al. (2013a) studies some benchmark cases. For example, in the limiting case $s \rightarrow 0$, i.e. with fishers changing their fishing strategy continuously, one gets $\dot{\pi}_i^*(t) = \pi_i^*(t)$, and the last equation in (1) can be replaced with

$$\begin{aligned} \dot{r}(t) &= r(t) [\pi_1^*(t) - (r(t)\pi_1^*(t) + (1 - r(t))\pi_2^*(t))] \\ &= r(t)(1 - r(t)) [\pi_1^*(t) - \pi_2^*(t)] \end{aligned} \quad (9)$$

which is the well-known replicator equation in continuous time (see again Weibull 1995; Hofbauer and Sigmund 1998), stating that $\dot{r}(t) > 0$ [$<$] if $\pi_1^*(t) > \pi_2^*(t)$ [$<$]. In this case, the model assumes the simpler form of a nonlinear 3-dimensional system of ordinary differential equations. In order to obtain analytical results, the case of constant prices is considered (i.e. $b_1 = b_2 = 0$). The dynamical model (1)

assumes the following simplified form:

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$$\begin{cases} \dot{X}_1 = X_1 \rho_1 \left(1 - \frac{X_1}{k_1}\right) - Nr \frac{a_1 X_1}{2\gamma_1} \\ \dot{X}_2 = X_2 \rho_2 \left(1 - \frac{X_2}{k_2}\right) - N(1-r) \frac{a_2 X_2}{2\gamma_2} \\ \dot{r} = r(1-r) \left[\frac{a_1^2 X_1}{4\gamma_1} - \frac{a_2^2 X_2}{4\gamma_2}\right] \end{cases} \quad (10)$$

Note that in the invariant subspaces defined by $r = 0$ and $r = 1$, the first two differential equations in (10) are uncoupled. The following propositions (proved in Bischi et al. 2013a) describe the steady states of the model and their local stability properties. To keep the notation short, in these propositions it is useful to define the aggregate parameters

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$$\alpha_1(r) = \rho_1 - \frac{a_1 Nr}{2\gamma_1} \text{ and } \alpha_2(r) = \rho_2 - \frac{a_2 N(1-r)}{2\gamma_2} \quad (11)$$

Proposition 1 (Boundary equilibria and their stability) For the model (10) the following statements hold:

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- the total extinction fixed points $E_r^0 = (0, 0, r)$, where $r \in [0, 1]$, are unstable non-hyperbolic nodes provided that $\alpha_i(r) > 0$, $i = 1, 2$;
- the equilibria with harvesting of only one species are given by:
 - $E_1^0 = (k_1, 0, 0)$ and $E_2^0 = (0, k_2, 1)$ (extinction of the harvested species), which are saddle points;
 - $E_1^1 = \left(0, k_2 \left(1 - \frac{Na_2}{2\gamma_2 \rho_2}\right), 0\right)$, if $Na_2 < 2\gamma_2 \rho_2$, and $E_2^1 = \left(k_1 \left(1 - \frac{Na_1}{2\gamma_1 \rho_1}\right), 0, 1\right)$, if $Na_1 < 2\gamma_1 \rho_1$ (extinction of the non-harvested species), which are saddle points;
 - $E_1^2 = \left(k_1, k_2 \left(1 - \frac{Na_2}{2\gamma_2 \rho_2}\right), 0\right)$ if $Na_2 < 2\gamma_2 \rho_2$ (both viable species with no harvesting of species 1), which is a stable node provided that

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$$a_1 < \sqrt{\frac{a_2^2 k_2 \gamma_1 (2\gamma_2 \rho_2 - Na_2)}{2k_1 \gamma_2^2 \rho_2}} \quad (12)$$

and a saddle point if the reverse inequality in (12) holds;

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- $E_2^2 = \left(k_1 \left(1 - \frac{Na_1}{2\gamma_1 \rho_1}\right), k_2, 1\right)$ if $Na_1 < 2\gamma_1 \rho_1$ (both viable species with no harvesting of species 2) which is a stable node provided that

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$$a_2 < \sqrt{\frac{a_1^2 k_1 \gamma_2 (2\gamma_1 \rho_1 - Na_1)}{2k_2 \gamma_1^2 \rho_1}} \quad (13)$$

and a saddle point if the reverse inequality in (13) holds.

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The next proposition characterizes an equilibrium with $r^* \in (0, 1)$, which is the case in which each species is always harvested by some fishers, of course with the restriction that each agent is allowed to fish just one species at each time.

Proposition 2 (Interior equilibrium and its stability) For the model (10) the following statements hold:

- There exists a unique interior equilibrium $E^* = (X_1^*, X_2^*, r^*)$ with $r^* \in (0, 1)$, where

$$X_i^* = \frac{a_j^2 k_1 k_2 \gamma_i (2a_2 \gamma_1 \rho_1 + 2a_1 \gamma_2 \rho_2 - a_1 a_2 N)}{2(a_2^3 k_2 \gamma_1^2 \rho_1 + a_1^3 k_1 \gamma_2^2 \rho_2)}, i = 1, 2; i \neq j \quad (14)$$

$$r^* = \frac{\gamma_1 \rho_1 (a_2^3 k_2 N \gamma_1 - 2a_2^2 k_2 \gamma_1 \gamma_2 \rho_2 + 2a_1^2 k_1 \gamma_2^2 \rho_2)}{N(a_2^3 k_2 \gamma_1^2 \rho_1 + a_1^3 k_1 \gamma_2^2 \rho_2)}$$

- Equilibrium biomass levels $X_i^* > 0, i = 1, 2$ iff $\alpha_i(r^*) > 0$, with $0 < r^* < 1$; this occurs in the following cases:
 - case 1: If $\alpha_1(1) > 0$ and $\alpha_2(0) > 0$ then the carrying capacity k_1 must satisfy

$$\hat{k}_1 = \frac{a_2^2 k_2 \gamma_1 (2\gamma_2 \rho_2 - Na_2)}{2a_1^2 \gamma_2^2 \rho_2} < k_1 < \frac{2a_2^2 k_2^2 \gamma_1^2 \rho_1}{2a_1^2 \gamma_1 \gamma_2 \rho_1 - Na_1^3 \gamma_2} = \bar{k}_1;$$

- case 2: If $\alpha_1(1) > 0$ and $\alpha_2(0) < 0$ then the carrying capacity k_1 must satisfy $0 < k_1 < \bar{k}_1$;
- case 3: If $\alpha_1(1) < 0$ and $\alpha_2(0) > 0$ then the carrying capacity k_1 must satisfy $k_1 > \hat{k}_1$;
- case 4: If $\alpha_1(1) < 0$ and $\alpha_2(0) < 0$ then it must be $\frac{2\gamma_1 \rho_1}{N} < a_1 < \frac{2a_2 \gamma_1 \rho_1}{Na_2 - 2\gamma_2 \rho_2}$, and at $a_1 = \frac{2a_2 \gamma_1 \rho_1}{Na_2 - 2\gamma_2 \rho_2}$ it is $X_1^* = X_2^* = 0$;
- finally, if $k_1 = \hat{k}_1$ then it is $r^* = 0$ and $E^* = E_1^2 = (k_1, k_2 (1 - N \frac{a_2}{2\gamma_2 \rho_2}), 0)$ whereas if $k_1 = \bar{k}_1$ then it is $r^* = 1$ and $E^* = E_2^2 = (k_1 (1 - N \frac{a_1}{2\gamma_1 \rho_1}), k_2, 1)$.

- If the equilibrium $E^* = (X_1^*, X_2^*, r^*)$ involves positive biomasses, then it is stable under the replicator dynamics in continuous time.

The cases discussed in the previous proposition help to understand how the interior equilibrium appears or disappears through contacts with border equilibria. In particular, the last two cases indicate that the interior equilibrium can have a contact with the extinction equilibrium E_r^0 or with the border equilibria E_1^2 and E_2^2 . It is also interesting to notice that, by Proposition 1, a single species, say species 1, does not become extinct provided that $\frac{2\gamma_1 \rho_1}{a_1 N} > r$, whereas by Proposition 2, the

condition $r^* < 1$ leads to $\frac{2\gamma_1\rho_1}{a_1N} < 1 + \frac{2a_2^2k_2\gamma_1^2\rho_1}{Na_1^3k_1\gamma_2}$. Therefore, if 199

$$\frac{2\gamma_1\rho_1}{a_1N} \in \left(r, 1 + \frac{2a_2^2k_2\gamma_1^2\rho_1}{Na_1^3k_1\gamma_2} \right) \tag{15}$$

then species 1 will survive both with an exogenous fixed r or with a r^* to which the continuous-time switching mechanism converges. However, if the fixed $r = \bar{r} \in (0, 1)$ is such that 200
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$$\frac{2\gamma_1\rho_1}{a_1N} \in (0, r) \tag{16}$$

i.e. too much harvesting pressure is imposed on species 1, then this fixed fraction of exploiters \bar{r} will lead the resource to extinction, whereas an endogenous r could avoid the occurrence of extinction for species 1 (the same reasoning applies, of course, to species 2). 203
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For the sake of comparison, the analytical results on the coexistence of both species can be summarized as follows: 207
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Corollary 1 *If $k_1, k_2 > 0$ and $\frac{2\rho_1\gamma_1}{Na_1} + \frac{2\rho_2\gamma_2}{Na_2} > 1$, the model with continuous replicator dynamics (10) converges to one of the following fixed points with coexistence of the two species:* 209
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- E_1^2 if $\frac{2\gamma_2\rho_2}{a_2N} \in \left(1 + \frac{2a_1^2k_1\gamma_2^2\rho_2}{Na_2^3k_2\gamma_1}, +\infty \right)$; 212
- E_2^2 if $\frac{2\gamma_1\rho_1}{a_1N} \in \left(1 + \frac{2a_2^2k_2\gamma_1^2\rho_1}{Na_1^3k_1\gamma_2}, +\infty \right)$; 213
- E^* whenever $\frac{2\gamma_1\rho_1}{a_1N} < 1 + \frac{2a_2^2k_2\gamma_1^2\rho_1}{Na_1^3k_1\gamma_2}$ and $\frac{2\gamma_2\rho_2}{a_2N} < 1 + \frac{2a_1^2k_1\gamma_2^2\rho_2}{Na_2^3k_2\gamma_1}$. 214

In the model (10) with the last differential equation dropped and the fraction r exogenously fixed to \bar{r} , if $k_1, k_2 > 0$ and $\frac{2\rho_1\gamma_1}{Na_1} + \frac{2\rho_2\gamma_2}{Na_2} > 1$, any $\bar{r} \in \left(1 - \frac{2\rho_2\gamma_2}{Na_2}, \frac{2\rho_1\gamma_1}{Na_1} \right)$ ensures the coexistence of both species. 215
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In short, if there is a coexistence equilibrium for the model (10) then there is at least an \bar{r} such that the model with an exogenously fixed r also converges to a coexistence equilibrium. On the contrary, if there exists an \bar{r} such that the model with exogenous r converges to a coexistence equilibrium, then the model with continuous replicator dynamics also converges to a coexistence equilibrium. 218
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The simpler specification considered above constitutes a useful benchmark for a deeper understanding of the full-fledged hybrid model. In fact, an equilibrium point for the system with continuous replicator (9) is also a fixed point for the hybrid system (1), although the converse is not necessarily true. This follows from the fact that the first and the second dynamic equations in the two specifications are identical, and the replicator dynamics in discrete and in continuous time share the same equilibrium conditions: $r(t) = r(t - s)$ for $r = 0, r = 1$ or $\pi_1^*(t) = \pi_2^*(t)$. In fact, if instantaneous profits are identical in equilibrium, then the average profits of 223
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the two strategies over the non-switching time interval of length s are also identical. 231
 Nonetheless, we can have an equilibrium point such that the average profits of 232
 the two strategies over the interval s are equal, even though instantaneous profits 233
 are not equal over the interval. In the case of the hybrid model (1), $r(t)$ becomes 234
 a piecewise-constant function, like an endogenously driven bang-bang variable, 235
 whose discontinuous jumps occur at discrete times and lead to sudden switches 236
 among different dynamic scenarios, which is typical behavior of hybrid systems.² 237
 Numerical simulations performed in Bischi et al. (2013a) allow to investigate the 238
 role played by s (the switching time) as well as the effects of non-constant prices, 239
 i.e. demand functions (3) with slope $b_i \neq 0$, $i = 1, 2$, on the dynamics of the 240
 more realistic hybrid model (1). Such analysis gives evidence of possible advantages 241
 of profit-driven self-regulated harvesting strategy choices over other practices, 242
 both from the point of view of biomass levels (i.e. biological sustainability) and 243
 wealth (economic profitability). Indeed, for proper sets of parameters, numerical 244
 simulations show that this kind of myopic and adaptive self-regulation may ensure 245
 a virtuous trade-off between profit maximization and resource conservation, driven 246
 by cost externalities and market pressure. 247

Other similar models can be proposed, as useful exercises, to describe situations 248
 with interacting species. In this case the two growth functions in (1) assume the 249
 more general forms $G_1(X_1, X_2)$ and $G_2(X_1, X_2)$, thus giving a further source of 250
 interdependence among the dynamic variables. Even if it is quite difficult to harvest 251
 only a single species when two fish populations interact in the same environment, 252
 the simulation of such a situation may provide useful theoretical information on 253
 the understanding of the trade-off between species interactions and endogenous 254
 evolutionary processes based on economic forces. For example, following the 255
 idea contained in Okuguchi (1984), in Bischi et al. (2013b) the case of two fish 256
 species which interact through a prey-predator relationship is considered. The time 257
 evolution of the fish biomasses is modelled by a system of differential equations 258
 known as Rosenzweig-MacArthur prey-predator model (see e.g. Rosenzweig and 259
 MacArthur 1963; Holling 1965; De Feo and Rinaldi 1997) with extra mortality 260
 terms both in prey and predator equations due to harvesting: 261

$$\begin{cases} \dot{X}_1 = X_1 G_1(X_1, X_2) - Nr(t)h_1^*(X_1, X_2) \\ \quad = X_1 \left[\rho \left(1 - \frac{X_1}{K} \right) - \frac{\beta X_2}{\alpha + X_1} \right] - Nr(t)h_1^*(X_1, X_2) \\ \dot{X}_2 = X_2 G_2(X_1, X_2) - N(1 - r(t))h_2^*(X_1, X_2) \\ \quad = X_2 \left(\frac{\eta \beta X_1}{\alpha + X_1} - d \right) - N(1 - r(t))h_2^*(X_1, X_2) \end{cases} \quad 262$$

²A comparison of models with endogenous switching among different regimes and with different time-scales is carried out in Bischi et al. (2014). A related hybrid evolutionary model for the adoption of technologies with different efficiency and environmental impact is considered in Lamantia and Radi (2015).

Also in this case, a simpler benchmark model is considered by assuming constant prices and a given fraction $r \in [0, 1]$ as an exogenous parameter. This leads to the following two-dimensional dynamical system:

$$\begin{cases} \dot{X}_1 = \rho X_1 \left(1 - \frac{X_1}{K}\right) - \frac{\beta X_1 X_2}{\alpha + X_1} - rN \frac{a_1 X_1}{2\gamma_1} \\ \dot{X}_2 = X_2 \left(\frac{\eta \beta X_1}{\alpha + X_1} - d\right) - (1-r)N \frac{a_2 X_2}{2\gamma_2} \end{cases} \quad (17)$$

The following characterization of equilibrium points holds (see Bischi et al. 2013b):

Proposition 3 *The dynamical system (17) has three non-negative equilibrium points, given by $S_0 = (0, 0)$, $S_1^r = \left(\frac{K(2\rho\gamma_1 - rNa_1)}{2\rho\gamma_1}, 0\right)$ and $S_2^r = (X_1^r, X_2^r)$, with*

$$X_1^r = \frac{\alpha(d + (1-r)N \frac{a_2}{2\gamma_2})}{\eta\beta - d - (1-r)N \frac{a_2}{2\gamma_2}}, X_2^r = \frac{(\alpha + X_1^r)}{\beta} \left[\rho - \frac{\rho X_1^r}{K} - rN \frac{a_1}{2\gamma_1}\right].$$

- *The Equilibrium S_1^r is positive if $2\rho\gamma_1 > rNa_1$, and S_2^r is positive provided that $\eta\beta > d + (1-r)N \frac{a_2}{2\gamma_2}$ and $X_1^r < \frac{K(2\rho\gamma_1 - rNa_1)}{2\rho\gamma_1}$.*
- *S_2^r becomes stable through a transcritical bifurcation at which S_1^r and S_2^r exchange stability, and it loses stability through a supercritical Hopf bifurcation; the analytical expressions for bifurcations curves are given by*

$$K = K_T^r = \frac{2\rho\gamma_1\alpha \left(d + (1-r) \frac{a_2 N}{2\gamma_2}\right)}{(2\rho\gamma_1 - Na_1) \left(\eta\beta - d - (1-r) \frac{a_2 N}{2\gamma_2}\right)} \text{ (Transcritical bifurcation curve)} \quad (18)$$

$$K = K_H^r = K_T^r + \frac{2\rho\gamma_1\alpha\eta\beta}{(2\rho\gamma_1 - Na_1) \left(\eta\beta - d - (1-r) \frac{a_2 N}{2\gamma_2}\right)} \text{ (Hopf bifurcation curve)} \quad (19)$$

The endogenous evolutionary dynamics of the fraction $r(t)$ can be introduced through two steps: first the case of continuous-time replicator dynamics is considered, modelled by (9); then the more general hybrid model obtained through discrete dynamics of $r(t)$ according to the third equation in (1) is addressed. The analytical and numerical results in Bischi et al. (2013b) show that this type of evolutionary mechanism may lead, also in this case, to a good compromise between profit maximization and resource conservation thanks to an evolutionary self-regulation based on cost and price externalities. In fact, the reduction of biomass of one species leads to increasing landing costs and it consequently favours the endogenous switching to the more abundant species; moreover, severe overfishing of one species causes decreasing prices and consequently decreasing profits.

The numerical results given in Bischi et al. (2013b) show that the endogenous switching mechanism, where fishers decide the variety to catch on the basis of their profits, leads to a high probability of coexistence of the two species provided that the dynamics of unexploited species converge to the stable coexistence equilibrium.

In this case, the endogenous switching mechanism significantly reduce the negative effects of exploitation. Another surprising characteristic of this endogenous switching is the reduction of the “oscillatory effect” due to oversupply of food. In fact, it is well known that, in a food-chain population model, the presence of self-sustained oscillations means oversupply of nutrients. In De Feo and Rinaldi (1997) some practical rules are given to reduce oscillations caused by overabundance of food at the bottom of the food chain. So, the exercise carried out in Bischi et al. (2013b) offers glimpse into the interesting properties of myopic and adaptive harvesting mechanisms driven by endogenous evolutionary processes. However this is just a starting point for further and deeper analysis. There are several aspects of the model that deserve to be explored more deeply. For example, the variable r , i.e. the fraction of fishers harvesting a given fish stock, is assumed to range in the interval $[0, 1]$, where 0 and 1 are always equilibria. When r converges to 0 or 1, one of the two species is no longer harvested and consequently it is not available in the market. This could be a reasonable practice only if the two species of fish are perfect substitute in consumers tastes (corresponding to the case $\sigma = 1$ in (3)); otherwise consumers may be heavily penalized in such equilibria. This may be avoided, for example, by introducing constraints on the dynamics of r . We refer to Bischi et al. (2013b) for further comments on this point.

3 Discrete-Time Fishery Models with Marine Protected Areas

Even if dynamic models in ecology have been traditionally formulated in continuous-time, discrete-time population dynamics have received a great amount of attention not only for the complex and intriguing time evolutions that they can produce even in the simplest systems, but also for relevant biological motivations that have been proposed to explain their usefulness in ecologic modeling. In fact, several authors stress that discrete-time population models should be used whenever reproduction happens at given breeding seasons, as several animal species successfully mate only during certain times of the year, thus giving non-overlapping generations (see e.g. Geritz and Kisdi 2004; May 1975). So, more and more discrete-time population models have been proposed in the literature (see e.g. Getz and Haight 1989; Cushing et al. 2004). A comparison between continuous-time and discrete-time dynamics in imperfectly competitive international commercial fisheries is given in Erjaee and Okuguchi (2006). A review of elementary fishery models in discrete time is given in Antonelli et al (2005), and their applications in the modelling of fishing regulation by the creation of marine protected areas is discussed in Antonelli et al. (2006), Bischi and Lamantia (2007), Bischi et al. (2009), and Sanchirico and Wilen (2001). International agreements for the creation of Marine Protected Areas (MPA) exist, and an increasing interest for this kind of regulation is witnessed by the rich literature, conferences and congresses devoted

to it (see e.g. Bohnsack 1993; Clark 1996; Sumaila 1998; Holland 2002; Sumaila 330
 and Charles 2002). As explained in Antonelli et al. (2006), the dynamic fishery 331
 models proposed in Bischi and Lamantia (2007) and Bischi et al. (2009), which 332
 will be described in this section, have been motivated by the project for the creation 333
 of a MPA in an international sea zone in the Adriatic Sea, in a region called “Fossa 334
 di Pomo”. In order to avoid strong opposition of fishers, this MPA is not intended 335
 as a reserve area (i.e. a no fishing zone) but it should be a region with carefully 336
 regulated fishing activity, for example where a limited constant-effort fishing is 337
 allowed. This may give rise to the possible fishers’ choice whether to harvest under 338
 an imposed limited effort, compulsory inside a MPA with abundance of fish stock, 339
 or without imposed limits (e.g. under oligopolistic competition), possible outside 340
 the MPA, i.e. in a zone with (likely) depleted fish stock. Of course, this choice 341
 is strongly influenced by the level of imposed fishing effort inside the MPA. On 342
 order to model this kind of fishery, in Bischi et al. (2009) an aquatic environment 343
 is assumed to be divided into two adjacent patches, characterized by different, 344
 and in some way complementary, fishing policies: an imposed *constant fishing* 345
effort in region 1 and *oligopolistic competition* in region 2, where fishers decide 346
 their harvesting on the basis of a boundedly rational “profit incursion” attitude. 347
 Moreover, given the total number of fishers, an adaptive switching mechanism is 348
 introduced, based on imitation dynamics (see e.g. Bischi et al. 2003; Vega-Redondo 349
 1996), that endogenously regulates the fraction of fishers that, at each time period, 350
 decide to operate in one of the two regions on the basis of observed profits. 351

Clearly the division in two different patches is a virtual one, in the sense that no 352
 physical boundaries exist, and fish can move among regions so that the quantity of 353
 biomass in one region depends not only on harvesting and biological growth in that 354
 region, but also on the stock and catch in the neighboring regions. The consequence 355
 is an interdependence which, at each time period, can increase the population of the 356
 zone where the fish density is smaller. 357

Let $X(t)$ be the total quantity of fish biomass in the water basin at time period 358
 t . We suppose that the water basin is virtually divided in two patches, labelled 359
 as patch 1 and patch 2. We denote by X_1 and X_2 the quantity of biomass in the 360
 different regions, with $X = X_1 + X_2$. We can imagine that a central authority 361
 draws a virtual line on the water basin so that it can impose a specific harvesting 362
 policy for agents operating in one of the two areas. Assuming that fish can 363
 migrate between the different patches according to a linear diffusion mechanism, 364
 the biological evolution of the resource in each patch is modelled by the following 365
 two-dimensional dynamical system: 366

$$\begin{cases} X_1(t+1) = X_1(t) G(X(t)) - \sigma[X_1(t) - X_2(t)] - H_1(t) \\ X_2(t+1) = X_2(t) G(X(t)) + \sigma[X_1(t) - X_2(t)] - H_2(t) \end{cases} \quad (20)$$

where $\sigma > 0$ is a diffusion coefficient, and $H_i(t)$ represents the quantity of fish 367
 harvested in time period t in region i . 368

A subdivision of a fishing region into patches, with these two kinds of fishing policies, is also considered in Bischi and Lamantia (2007). However, the division modelled in that paper is markedly different, because two independent growth functions are considered, in the sense that the fish population of each region is assumed to interact only with individuals of the same region. Instead, in this case the growth function in each patch involves interaction with both subpopulations, i.e. summing up the two Eqs. (20) we have

$$(X_1 + X_2)(t + 1) = (X_1(t) + X_2(t))G(X_1(t) + X_2(t)) - H(t)$$

where $H(t) = H_1(t) + H_2(t)$ is the total harvested quantity.

In the following, a logistic growth function is considered, with intrinsic growth rate α and carrying capacity $K = \alpha/\beta$, i.e.

$$X(t + 1) = X(t)G(X(t)) = X(t)(1 + \alpha - \beta X(t))$$

In order to derive the two different harvesting functions in patch 1 and patch 2 let us assume that in patch 1 n_1 fishers are allowed to harvest with an imposed constant fishing effort, whereas in patch 2 n_2 fishers are engaged in a Cournot oligopolistic competition. Let $N = n_1 + n_2$ be the total number of fishers. We denote by $r = n_2/N$ the fraction of fishers that decide to harvest outside the MPA, choosing the quantity to harvest according to a profit-increasing Cournot oligopoly strategy, as specified below. Of course $(1 - r)$ represents the fraction of agents fishing in region 1, i.e. inside the MPA.

If E is the individual fishing effort imposed by a regulator in the MPA, denoted as patch 1, then

$$H_1(X_1) = qEN(1 - r)X_1.$$

where q is the ‘‘catchability’’ coefficient, linked to the adopted technology. The idea of a constant fishing effort is very common in the literature on mathematical bioeconomic modelling of fisheries (see e.g. Clark 1990 and references therein) and constitutes the most employed method of control in fisheries.

In patch 2 fishers engage in a Cournot oligopolistic competition, i.e. they decide, at each time period, their harvesting quantity according to profit-increasing behavior taking into account economic externalities. Here we assume the existence of a unique market for the harvested resource determined by the linear inverse demand function

$$p(t) = a - bH(t) \tag{21}$$

where a and b are positive parameters. Notice that we implicitly assumed that the total harvested quantity $H(t)$ is entirely sold in the market.

The profit at time t of a representative fisher operating in zone 1 is given by 403

$$\begin{aligned} \pi_1(t) &= qEX_1(t)(a - bH(t)) - \delta E & (22) \\ &= qEX_1(t) \{a - b [ENq(1 - r(t))X_1(t) + Nr(t)x(t)]\} - \delta E \end{aligned}$$

where δ is the unitary cost of effort, $H_1(t) = N[1 - r(t)]qEX_1(t)$ and $H_2 = 404$
 $Nr(t)x(t)$ represent the total harvesting in patch 1 and 2 respectively, $r(t)$ is the 405
fraction of agents operating outside the MPA in period t , $x(t)$ is their individual 406
catch, whose determination is given below. 407

The cost function of a representative fisher that chooses to fish in zone 2, for 408
harvesting a quantity x when the fish stock is X_2 , is given by 409

$$C(x, X_2) = c + \gamma \frac{x^2}{X_2} \quad 410$$

where γ reflects the inefficiency for harvesting outside the MPA and c is a fixed 411
cost, which includes also the effort for solving the harvesting decision problem. 412
The quantity of resource harvested by a single oligopolist, $x(t)$, is determined in the 413
following way. Assuming that all fishers are homogeneous, we can write the time t 414
profit of a representative fisher working in zone 2 as 415

$$\pi_2(t) = x(t) (a - bH) - \gamma \frac{x(t)^2}{X_2(t)} - c \quad (23)$$

Due to the complexity of the underlying model, it could be difficult for an oligopolist 416
to compute the profit maximizing level of harvesting. For this reason, we assume 417
that oligopolists act in a “boundedly rational” way, in the sense that they update 418
their harvesting plans in the direction of increasing profits, as specified by the so 419
called profit-gradient dynamics (see Flam 1993; Furth 1986; Bischi and Naimzada 420
2000). Thus, the quantity $x(t + 1)$ of a representative oligopolist as a function of 421
 $r(t)$, $x(t)$, $X_1(t)$ and $X_2(t)$ can be written as 422

$$\begin{aligned} x(t + 1) &= x(t) + k \cdot \frac{\partial \pi_2(t)}{\partial x(t)} = & (24) \\ &= x(t) + k \cdot \left\{ a - bN [2r(t)x(t) + Eq(1 - r(t))X_1(t)] - \frac{2x(t)\gamma}{X_2(t)} \right\} \end{aligned}$$

where $k > 0$ is the speed of adjustment to the direction of the profit gradient. 423

Agents can decide at any time period the patch to fish in and, consequently, 424
the harvesting strategy, i.e. whether to harvest the resource according to free 425
competition outside the MPA, following a Cournot oligopolistic game, or through 426
(controlled) constant effort landing inside the MPA. Since agents do not have full 427
knowledge of the underlying structure of the game, the switching mechanism is 428

assumed to be governed by an imitative behavior.³ Again, a replicator dynamics 429
 may be used (see e.g. Weibull 1995; Vega-Redondo 1996; Hofbauer and Sigmund 430
 1998) where at each time period any fisher samples an agent that has chosen a 431
 different strategy in the past, and switches to the other strategy if the profit of the 432
 sampled agent has been greater than his own. However, in Bischi et al. (2009) the 433
 probability to change behavior is modelled according to a different kind of imitative 434
 behavior, more similar to word of mouth dynamics (see Bischi et al. 2003), based 435
 on the following switching probabilities 436

$$p_{O \rightarrow E} = (1 - r)\mathbb{P}(\pi_1 > \pi_2) ; p_{O \rightarrow O} = 1 - p_{O \rightarrow E}$$

$$p_{E \rightarrow O} = r\mathbb{P}(\pi_2 > \pi_1) ; p_{E \rightarrow E} = 1 - p_{E \rightarrow O}$$

where “ $p_{O \rightarrow E}$ ” is the probability to change from strategy “ O ” (“Oligopoly”) to 437
 strategy “ E ” (constant effort). This probability is given by the product between the 438
 probability to meet an agent harvesting with constant effort, i.e. $(1 - r)$, and the 439
 probability that the profit of that agent is greater than the profit for a representative 440
 oligopolist, that is $\mathbb{P}(\pi_1 > \pi_2)$; a similar definition holds for the other quantities, 441
 with an obvious meaning of the symbols. The cumulative distribution (probability) 442
 function $\mathbb{P}(\pi_1 > \pi_2) = \Psi(\pi_1 - \pi_2)$ is monotone increasing with $\Psi(-\infty) = 0$ and 443
 $\Psi(+\infty) = 1$. Of course, $\mathbb{P}(\pi_2 \geq \pi_1) = 1 - \Psi(\pi_1 - \pi_2)$. 444

All in all, the dynamic (deterministic mean field) equation describing how the 445
 expected fraction of oligopolists evolves is given by 446

$$r(t + 1) = r(t) + [1 - r(t)]p_{E \rightarrow O} - r(t)p_{O \rightarrow E}$$

$$= r(t) + [1 - r(t)]r(t) [1 - \Psi(\pi_1(t) - \pi_2(t))] - r(t)(1 - r(t))\Psi(\pi_1(t) - \pi_2(t))$$

$$= r(t) [1 - (1 - r(t))M(\pi_1(t) - \pi_2(t))] \quad (25)$$

where $M(\cdot) = 2\Psi(\cdot) - 1$. Equation (25) can be interpreted as a balance equation, 447
 saying that the fraction of agents that at time $t + 1$ operates outside the MPA, $r(t +$ 448
 $1)$, is increased by the expected fraction of agents that decide to exit the MPA, 449
 $[1 - r(t)]p_{E \rightarrow O}$, and decreased by agents entering the MPA, i.e. $r(t)p_{O \rightarrow E}$. 450

The main properties that the function M inherits from the commonly used 451
 cumulative distributions Ψ are: (1) $M(0) = 0$; (2) $\lim_{z \rightarrow -\infty} M(z) = -1$; (3) 452
 $\lim_{z \rightarrow +\infty} M(z) = 1$; (4) M is an odd function and (5) M is convex in $(-\infty, 0)$ and 453
 concave in $(0, +\infty)$. 454

In the following, we explicitly assume the functional form 455

$$M(z) = \frac{2}{\pi} \arctan\left(\frac{\lambda\pi}{2}z\right) \quad 456$$

³A non-evolutionary version of this model has been addressed in Lamantia and Sbragia (2006).

where the parameter $\lambda = M'(0) = \frac{1}{2}\Psi'(0) > 0$ represents the agents' propensity to strategy switching. 457
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All in all, the dynamical system obtained by considering the natural resources evolution in Eqs. (20) with $H_1(t) = N[1 - r(t)]qEX_1(t)$ and $H_2(t) = Nr(t)x(t)$, individual oligopolistic harvesting in (24), and expected fraction of oligopolistic competitors (25), is described by the following map in \mathbb{R}^4 with dynamic variables X_1, X_2, x and r : 459
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$$T : \begin{cases} X_1(t+1) = X_1(t)G(X_1(t) + X_2(t)) - \sigma[X_1(t) - X_2(t)] - N[1 - r(t)]qEX_1(t) \\ X_2(t+1) = X_2(t)G(X_1(t) + X_2(t)) + \sigma[X_1(t) - X_2(t)] - Nr(t)x(t) \\ x(t+1) = \max \left[0, x(t) + k \cdot \left\{ a - bN[2r(t)x(t) + qE(1 - r(t))X_1(t)] - \frac{2x(t)\gamma}{X_2(t)} \right\} \right] \\ r(t+1) = r(t) \left\{ 1 - (1 - r(t)) \frac{2}{\pi} \arctan \left[\frac{\lambda\pi}{2} (\pi_1(t) - \pi_2(t)) \right] \right\} \end{cases} \quad (26)$$

It is important to note that the max operator prevents that oligopolists can choose negative quantity of harvesting. The sets $r = 0$ and $r = 1$ are dynamic invariant, in the sense that if $r(t) = 0$ (1) then also $r(t+1) = 0$ (1). This corresponds to the obvious statement that when a strategy is not performed at a given time it can not be imitated. When $r = 0$, i.e. all agents exert constant effort, the model reduces to a two-dimensional map, as the equations for x and r in (26) are redundant. 464
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A steady state of the dynamical system (26) is a point $S = (X_1^*, X_2^*, x^*, r^*)$ satisfying the following nonlinear system of equations 470
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$$\begin{cases} X_1[\alpha - \beta(X_1 + X_2)] - \sigma(X_1 - X_2) - N(1 - r)qEX_1 = 0 & (1) \\ X_2[\alpha - \beta(X_1 + X_2)] + \sigma[X_1 - X_2] - Nr x = 0 & (2) \\ aX_2 - bNX_2[2rx + qE(1 - r)X_1] - 2x\gamma = 0 & (3) \\ r(1 - r)(\pi_1 - \pi_2) = 0 & (4) \end{cases} \quad (27)$$

where π_1 and π_2 are given in (22) and (23) respectively. When no biomass is present in each patch, i.e. $X_1 = X_2 = 0$, from (27, 2) we get $x = 0$ and (27, 4) reduces to $r(1 - r)(c - \delta E) = 0$. In this case equilibria are given by $S_1^0 = (0, 0, 0, 0)$ and $S_2^0 = (0, 0, 0, 1)$. When $c = \delta E$, any level $r^* \in [0, 1]$ guarantees an equilibrium of the type $S_3^0 = (0, 0, 0, r^*)$. It is easy to prove (see Bischi et al. 2009) that all these boundary equilibria S_1^0, S_2^0, S_3^0 are unstable for any parameters' values. Indeed, all these cases represent extinction equilibria as they involve no biomass in each patch. 472
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Let us now consider the case $r = 0$, i.e. all agents harvesting inside the MPA. Obviously Eq.(27, 4) is satisfied and from (27, 3) we get that the harvesting equilibrium level of each representative agent is given by $x_0^* = \frac{(a - bNqEX_1)X_2}{2\gamma}$. Note that as all agents harvest in this case inside the MPA, the equilibrium level x_0^* has no practical relevance whatsoever. Hence system (27) reduces now to 479
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$$\begin{cases} X_1[\alpha - \beta(X_1 + X_2)] - \sigma(X_1 - X_2) - NqEX_1 = 0 \\ X_2[\alpha - \beta(X_1 + X_2)] + \sigma[X_1 - X_2] = 0 \end{cases} \quad 484$$

From the second equation, for $X_2 \neq \frac{\alpha}{\beta}$, we obtain $X_1 = g(X_2) = \frac{X_2(\alpha - \sigma - \beta X_2)}{\beta X_2 - \sigma} > 0$ 485
 that substituted back in the first equation gives the equilibrium level in the variable 486
 X_2 as a zero of the single variable function 487

$$f(X_2) = \frac{X_2 [(2X_2\beta - \alpha)(\alpha - 2\sigma)\sigma + ENq(\sigma - X_2\beta)(\alpha - \sigma - X_2\beta)]}{(\sigma - X_2\beta)^2} \quad 488$$

Clearly, it results that $f(0) = 0$ and $\lim_{X_2 \rightarrow +\infty} f(X_2) = +\infty$; moreover, f has a vertical 489
 asymptote at $X_2 = \frac{\sigma}{\beta}$, as $\lim_{X_2 \rightarrow \frac{\sigma}{\beta}} f(X_2) = -\infty$. By continuity of f in the interval 490
 $(\frac{\sigma}{\beta}, +\infty)$, at least one $X_2^* > \frac{\sigma}{\beta}$ exists such that $f(X_2^*) = 0$, hence defining the 491
 equilibrium point $S_0^* = (g(X_2^*), X_2^*, x^*, 0)$. Note that condition $f'(0) > 0$, equivalent 492
 to $\frac{\alpha}{\sigma} < 1 + \frac{1}{1+ENq}$, implies the existence of another equilibrium point such that 493
 $X_2^* \in (0, \frac{\sigma}{\beta})$. 494

The analysis of the case $r = 1$, i.e. with all agents harvesting outside the 495
 MPA, leads to a similar exercise. Again Eq. (27, 4) is satisfied and the harvesting 496
 equilibrium level of each representative agent is $x_1^* = \frac{\alpha X_2}{2(bNX)} > 0$. The existence of 497
 equilibria can be studied similarly to the previous case. Under the likely assumption 498
 that $\alpha > 2\sigma$ it is possible to prove analytically that no equilibrium exists with 499
 $X_1^* < \frac{\sigma}{\beta}$ and at least one equilibrium $S_1^* = (X_1^*, g(X_1^*), x_1^*, 1)$ always exists with 500
 $X_1^* > \frac{\sigma}{\beta}$. Moreover, it is easy to verify that condition $a < \frac{4b\gamma\sigma(\alpha-2)}{\beta}$ ensures 501
 uniqueness of equilibrium S_1^* under the case $r = 1$. 502

Now we tackle the problem of existence of interior equilibria with $r \in (0, 1)$ and 503
 with $X_1, X_2 > 0$. As before, we can find equilibria in terms of a two-dimensional 504
 system of equations. In fact, from (27, 1) we can write 505

$$r(X_1, X_2) = \frac{ENq - \alpha + \beta(X_1 + X_2) - \sigma(X_1 - X_2)}{ENq}. \quad 506$$

By adding together (27, 1) and (27, 2) and substituting back the expressions for r 507
 and $1 - r$, an expression for $x(X_1, X_2) = \frac{[(\alpha - \beta(X_1 + X_2))X_2 + \sigma(X_1 - X_2)]EqX_1}{X_1[ENq - \alpha + \beta(X_1 + X_2)] - \sigma(X_1 - X_2)}$ is obtained. 508
 By employing the previous expressions of $r(X_1, X_2)$ and $x(X_1, X_2)$ in (27, 3) and (27, 509
 4), the following system of equations is obtained, in which only the variables X_1 and 510
 X_2 appear, which characterize interior equilibria with $r \in (0, 1)$ and $X_1, X_2 > 0$: 511

$$\begin{cases} a - bNqE(1-r)X_1 - 2\left(bNr + \frac{\gamma}{X_2}\right)x = 0 \\ qEX_1[a - b(qENX_1(1-r))] - \delta E + c = x\left[a + bNqEX_1(2r-1) - \left(bNr + \frac{\gamma}{X_2}\right)x\right] \end{cases} \quad 512$$

where the last equation can be referred to as the locus of points where profits inside 513
 and outside the MPA are equal. Note that any solution (X_1^*, X_2^*) only define a 514
 feasible equilibrium point when $r(X_1^*, X_2^*) \in [0, 1]$ and $x(X_1^*, X_2^*) > 0$. 515

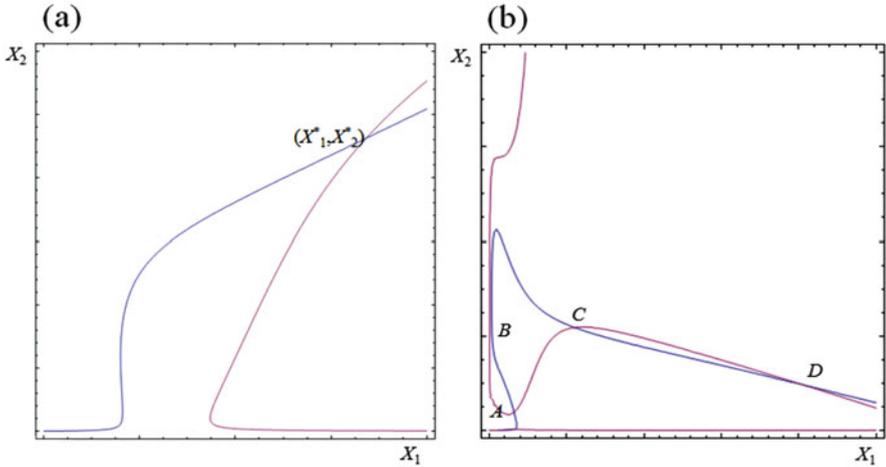


Fig. 1 (a) Unique interior equilibrium of system (27) as intersection point of the depicted curves; (b) multiple interior equilibria of system (27)

This system can be solved numerically. An easy situation is depicted in Fig. 1a, 516
 with parameters $\alpha = 2$; $\beta = 0.2$; $\sigma = 0.6$; $E = 1$; $N = 80$; $q = 0.1$; $a = 2$; 517
 $b = 0.3$; $\gamma = 0.5$; $\delta = 0.5$; $c = 0.5$, where the unique positive equilibrium with 518
 $(X_1^*, X_2^*) \simeq (6.7341, 4.6296)$ is represented as the intersection point of two curves 519
 defined by the previous system. It can be easily checked that the corresponding 520
 equilibrium level of r and x are also feasible. More involved equilibrium curves 521
 are depicted in Fig. 1b, where parameters are set as $\alpha = 3$; $\beta = 0.4$; $\sigma = 0.3$; 522
 $E = 2$; $N = 80$; $q = 0.1$; $a = 2$; $b = 0.3$; $\gamma = 0.5$; $\delta = 0.5$; $c = 0.5$. In this 523
 second example, four equilibria, A, B, C, D, are obtained (also in these cases the 524
 corresponding equilibrium level of r and x are feasible). 525

As stability analysis for equilibria of the type S_0^* and S_1^* , as well as for interior 526
 equilibria with $r \in (0, 1)$, is not an easy task, only some numerical simulations are 527
 proposed in Bischi et al. (2009), from which the following economic interpretations 528
 are obtained. When the regulator is able to enforce low effort levels in the MPA, then 529
 the system is characterized by stability in all relevant quantities: biomass in each 530
 patch, harvesting and fraction of oligopolists. This is a consequence of imposing low 531
 harvesting levels in the first patch so that many agents are better off by operating in 532
 patch 2. Indeed, patch 1 serves as a restocking area, since harvesting levels are kept 533
 at a minimum. However, if the regulator let agents increase their catches in patch 1, 534
 then a larger fraction of fishers tends to switch to the first patch. As a consequence, 535
 also individual harvesting in patch 2 is increased, in order to compensate higher 536
 harvesting (and profit) from patch 1. This mechanism becomes more complicated 537
 when a Neimark-Sacker bifurcation of the interior equilibrium occurs, after which 538
 persistent endogenous oscillations in state variables (biomasses, harvesting, and 539
 fraction of oligopolists) are observed. These effects are more evident with higher 540

agents' propensity to switch strategies, with consequent more persistent oscillations 541
in biomass levels and fractions of oligopolists. Similar results can be obtained, for 542
a given level of imposed efforts, by varying the cost of effort δ . In particular, we 543
observe that the higher the cost of effort is, the more stable the system appears. 544
Moreover, the oscillatory behavior encountered with low costs of effort disappears 545
as δ is increased over a given value. 546

The effect of changing c , the fixed costs of a representative oligopolist, is also 547
considered. This fixed cost can also be interpreted as a fixed tax imposed to each 548
oligopolist at each time period. In this case, as before, there is a range of fixed costs 549
ensuring stability of the steady state but now loss of stability of the steady state 550
can happen through flip (period doubling) bifurcations, and when c is above a given 551
level, exploiting patch 2 gives rise to sure losses so that almost all agents operate 552
in patch 1. Hence patch 2 tends to become a no-take area. In this scenario we have 553
that $r \rightarrow 0$ as time passes and the third equation in (26), representing harvesting of 554
a representative oligopolist, does not influence biomass levels in practice. 555

To summarize, the model with two different kinds of fishing strategies reveals a 556
much broader array of dynamical behavior with respect to those obtained in the 557
system with a homogeneous fishing environment: besides steady state behavior, 558
periodic, quasi-periodic and chaotic dynamics emerge, and qualitative changes (or 559
bifurcations) are detected as some parameters of the model are varied. The main 560
conclusions are related to the sustainability of exploitation in the whole region. In 561
fact, sustainability is favored by reducing the harvesting levels in one of the two 562
patches, which hence serves as a restocking area. 563

The important role of marine protected areas can be deduced from the results 564
shown in Bischi et al. (2009), where it is shown that low levels of effort tend to 565
stabilize the system around equilibrium levels, whereas large variability of state 566
variables are present when effort is over some threshold values. This effect is more 567
evident as agents' propensity to switch strategy is increased. Stability of the state 568
variable is important as it implies also stability in profits. In any case, as fishing 569
effort is increased more variability in profits for oligopolists than for constant- 570
effort exploiters is observed; hence oligopolistic behavior can be very risky when 571
the MPA is not properly enforced. This point suggests to keep regulated effort at 572
low levels not only to prevent overfishing inside the MPA, but also to reduce the 573
variability of oligopolists' profits. Moreover the regulator can influence the fishing 574
level outside the MPA by properly tuning the parameter c , that is interpretable as a 575
tax on oligopolists. 576

A peculiar feature that can be deduced from the numerical results shown in 577
Bischi et al. (2009) is the tendency to produce oscillatory time patterns, with 578
periodic, quasi-periodic or chaotic oscillations. In a real system this property may 579
induce some difficulties in the analysis of empirical data when policy makers try to 580
understand the trend of available fish stock after the creation of a MPA. In other 581
words, the benefits of the creation of a MPA can be properly evaluated only if 582
data is observed for a long period after the MPA is established. Of course, the 583
model suggested in Bischi et al. (2009) provides a very simplified and stylized 584
representation of the underlying real system. In fact, the presence of many important 585

factors have been neglected, such as the presence of different species of fish as well as the existence of different age (or size) classes. This point is especially important in the particular case that motivated the paper Bischi et al. (2009), the project of a MPA in the region known as Fossa di Pomo in the central Adriatic Sea. In fact, the importance of this region is related to the presence of young (hence small) fishes (it is denoted as a “nursery”). So, the limitation of fishing activity in that region has the important effect of increasing the number of big size fishes around it. This effect, as well as other boundary effects around a MPA, has not been dealt with in Bischi et al. (2009). Moreover, also the effects of the mobility of fishes different from linear diffusion due to density differences have been neglected. In other words, the model proposed in Bischi et al. (2009) constitutes a first step towards more advanced models, which can be obtained by including more realistic assumptions. Of course, the inclusion of such assumptions has the consequence of yielding a more complicated mathematical structure to the model, and the usual trade-off between mathematical simplicity and a realistic description of real systems always constitutes a difficult and challenging issue.

4 Heterogeneous Harvesting Rates with Punishment

In this section we illustrate a dynamic model proposed in Bischi et al. (2004) to describe an open access fishery exploited by a population of interacting agents that sell the harvested resource in the same market. This population of harvesters is divided into two groups: cooperators, that decide their harvesting policy by maximizing the overall profit of their group, and defectors, that just maximize their own profit. Defectors are punished by sanctions (or taxes or social disapproval) imposed by cooperators. An evolutionary mechanism, based on the replicator dynamics, is introduced to model the time changes in the proportions of defectors and cooperators within the population, and this leads to the qualitative study of a two-dimensional nonlinear dynamical system that describes the time evolution of the resource stock and the population share between cooperators and defectors.

Let us assume that a population of n agents harvests from a common property renewable resource stock X , and sells the harvested resource at a price p determined by the total harvested quantity according to a given linear demand function

$$p = a - bH \tag{28}$$

where a and b are positive constants. H is the total harvesting, which is entirely supplied and sold in the market:

$$H = \sum_{i=1}^{ns} x_c^i + \sum_{i=1}^{n(1-s)} x_d^i \tag{619}$$

where $0 \leq s \leq 1$ is the fraction of “cooperators” that form a coalition (a cooperative venture) and consequently each of them decides to harvest a quantity x_c^i , $i = 1, \dots, ns$, that maximizes the overall profit of the coalition. Agents in remaining fraction $(1 - s)$ behave as “selfish” profit maximizers and are denoted as “defectors” (with respect to the socially more desirable cooperative behavior). Each of them harvest a quantity x_d^i , $i = 1, \dots, n(1 - s)$, that maximize his/her own profit.

Player’s i cost function for harvesting a quantity x when a fish stock X is present is given by the usual expression

$$C(x, X) = \gamma \frac{x^2}{X}. \tag{29}$$

Following Sethi and Somanathan (1996), an extra-cost is considered, due to the presence of social norms that are intended to punish fishers that behave as defectors, i.e. self-interested profit maximizers without any care of social optimum. Like in Sethi and Somanathan (1996) cooperators are entrusted to punish defectors by applying sanctions. This may be done directly by exerting a direct punishment, such as social disapproval damage or physical damage or destruction of equipment, as observed in less developed societies, or by alerting authorities so that they can impose sanctions according to the laws in force. Such punishment is costly for the defectors, the cost being $ns\xi$, where ξ is the amount of the sanction and ns represents the probability that a defector is notified by a cooperator. However, in general, this kind of punishment is also costly for the cooperators, the cost being proportional to the number of defectors. Let $n(1 - s)\psi$ be this extra cost for cooperators (of course $\xi > \psi$, and we shall often consider $\psi = 0$ in the following). All in all, the profit of the i -th cooperator is

$$\pi_c^i = x_c^i (a - bH) - \gamma_c \frac{(x_c^i)^2}{X} - n(1 - s)\psi \tag{30}$$

where γ_c measures the fishing inefficiency of cooperators and $n(1 - s)\psi$ represents the cost that cooperators have to face in order to punish defectors. The profit of the i -th defector is

$$\pi_d^i = x_d^i (a - bH) - \gamma_d \frac{(x_d^i)^2}{X} - ns\xi \tag{31}$$

where γ_d measures the fishing inefficiency of defectors and $ns\xi$ represents the punishment that defectors have to bear for causing the negative externality in the community.

Each cooperator determines x_c^i by solving the optimization problem

$$\max_{x_c^i} \pi^V = \max_{x_c^i} \sum_{i=1}^{ns} \pi_c^i \tag{32}$$

where π^V , which is a concave function in the variables x_c^i , denotes the total profit of the cooperative venture. Assuming interior optimum, the first order conditions give a system of linear equations in the unknowns x_c^i

$$\frac{\partial \pi^V}{\partial x_c^i} = a - 2b \sum_{k=1}^{ns} x_c^k - b \sum_{k=1}^{n(1-s)} x_d^k - \frac{2\gamma_c}{X} x_c^i = 0 \quad (33)$$

Each defector determines x_d^i by solving the optimization problem

$$\max_{x_d^i} \pi_d^i \quad (34)$$

Assuming, again, interior optimum, the first order conditions give a system of linear equations in the unknowns x_d^i

$$\frac{\partial \pi_d^i}{\partial x_d^i} = a - b \sum_{k=1}^{sn} x_c^k - b \sum_{k=1}^{(1-s)n} x_d^k - b x_d^i - \frac{2\gamma_d}{X} x_d^i = 0 \quad (35)$$

Equations (33) and (35) constitute a linear system of n equations with n unknowns. However, it is straightforward to see that any cooperator faces the same optimization problem, and analogously for defectors. In fact, if we denote by $x_c^{TOT} = \sum_{k=1}^{sn} x_c^k$ the total harvest of cooperators and by $x_d^{TOT} = \sum_{k=1}^{(1-s)n} x_d^k$ the total harvest of defectors, from (33) we get

$$x_c^i = \frac{X}{2\gamma_c} (a - 2b x_c^{TOT} - b x_d^{TOT}) \quad \forall i = 1, \dots, sn \quad (36)$$

and from (35) we get

$$x_d^i = \frac{X}{bX + 2\gamma_d} (a - b(x_c^{TOT} + x_d^{TOT})) \quad \forall i = 1, \dots, (1-s)n \quad (37)$$

Thus, denoting the optimal harvesting decision of each cooperator by x_c and the optimal harvesting decision of each defector by x_d , these quantities can be obtained by solving the two linear equations

$$\begin{cases} a - 2(bsn + \frac{\gamma_c}{X})x_c - b(1-s)nx_d = 0 \\ a - bsnx_c - [b(1 + (1-s)n) + \frac{2\gamma_d}{X}]x_d = 0 \end{cases} \quad (38)$$

from which the two linear *reaction functions* are obtained

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$$\begin{aligned}
 x_c = h_c(x_d) &= \frac{aX}{2(bsnX + \gamma_c)} - \frac{b(1-s)nX}{2(bsnX + \gamma_c)}x_d \\
 x_d = h_d(x_c) &= \frac{aX}{b(1 + (1-s)n)X + 2\gamma_d} - \frac{bsnX}{b(1 + (1-s)n)X + 2\gamma_d}x_c
 \end{aligned}
 \tag{36}$$

These reaction functions allow one to compute, respectively, the optimal harvesting decision of a “representative cooperator”, given the harvesting decision of a representative defector, and the optimal harvesting decision of a “representative defector”, given the harvesting decision of a representative cooperator. These two reaction functions always intersect in a unique point (x_d^*, x_c^*) , whose coordinates are given by

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$$x_d^*(X, s) = aX \frac{bsnX + 2\gamma_c}{b^2sn(n(1-s) + 2)X^2 + 2b(2ns\gamma_d + \gamma_c + \gamma_cn(1-s))X + 4\gamma_c\gamma_d}
 \tag{37}$$

and

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$$x_c^*(X, s) = \frac{X}{2} \frac{a - b(1-s)nx_d^*(X, s)}{bsnX + \gamma_c}.
 \tag{38}$$

For each $s \in [0, 1]$ both x_d^* and x_c^* are increasing functions of X that vanish at $X = 0$, where they have slopes $\frac{\partial x_d^*(X, s)}{\partial X}|_{X=0} = \frac{a}{2\gamma_d}$ and $\frac{\partial x_c^*(X, s)}{\partial X}|_{X=0} = \frac{a}{2\gamma_c}$, respectively. Moreover, both x_d^* and x_c^* tend to saturate as $X \rightarrow +\infty$, at the values

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$$x_d^*(+\infty, s) = \frac{a}{b[(1-s)n + 2]} \quad \text{and} \quad x_c^*(+\infty, s) = \frac{a}{bsn[(1-s)n + 2]}
 \tag{39}$$

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respectively. The gap between x_d^* and x_c^* , for large values of the fish stock, increases with increasing prices and with the number ns of cooperators, being

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$$x_d^*(+\infty, s) - x_c^*(+\infty, s) = \frac{a}{b[(1-s)n + 2]} \left(1 - \frac{1}{sn}\right)
 \tag{40}$$

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The total harvesting at the Nash equilibrium between the two groups, given by

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$$H^*(X, s) = n[sx_c^*(X, s) + (1-s)x_d^*(X, s)]
 \tag{39}$$

is an increasing and concave function with respect to X , such that $H^*(0, s) = 0$,

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$$\frac{\partial H^*(X, s)}{\partial X}|_{X=0} = \frac{na}{2} \left(\frac{s}{\gamma_c} + \frac{1-s}{\gamma_d}\right)
 \tag{41}$$

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and for $X \rightarrow \infty$ it saturates at the value

$$H^*(+\infty, s) = \frac{a}{b} \left(1 - \frac{1}{2 + (1-s)n} \right)$$

The properties of the harvesting function H^* are similar to the ones assumed in Sethi and Somanathan (1996), but in this case the harvesting function is derived from an explicit underlying optimization problem. This implies that the effects of the economic and biologic parameters can be unequivocally studied. First of all we notice that for $sn \geq 2$ (i.e. if at least two cooperators exist) it is $x_c^* < x_d^*$ for large values of fish stock, whereas for small values of X the comparison between x_d^* and x_c^* depends on the respective cost parameters. It is also worth to notice that both x_d^* and x_c^* increase if the selling price increases, i.e. a increases and/or b decreases in the demand function (28). Notice also that $H^* < a/b$ for each $X > 0$, so that prices are always positive. Moreover, if s is decreased, i.e. the number of defectors is increased, then $H^*(+\infty, s)$ increases. In other words, as expected, in the presence of abundant resource the total harvest is greater if the number of defectors increases. In the limiting case $s = 0$ (all defectors) we have $H^*(+\infty, 0) = \frac{a}{b} \left(1 - \frac{1}{2+n} \right)$, and in the opposite limiting case $s = 1$ (all cooperators, equivalent to the case of a sole owner) we get $H^*(+\infty, 1) = a/2b$, the monopolist optimal harvesting.

These results allow us to compute, at the Nash equilibrium, the profit of a representative defector and that of a representative cooperator, given by

$$\pi_d^* = x_d^* (a - bH^*) - \gamma_d \frac{(x_d^*)^2}{X} - ns\xi, \quad \pi_c^* = x_c^* (a - bH^*) - \gamma_c \frac{(x_c^*)^2}{X} - n(1-s)\psi \tag{40}$$

respectively. Plugging the expressions of x_c^* , x_d^* and H^* into (40), π_d^* and π_c^* and can be rewritten as:

$$\pi_d^* = \frac{a^2X(bnsX + 2\gamma_c)^2 (bX + \gamma_d)}{[bX (bn (-n(1-s) - 2)) sX - 2bX (n(1-s) + 1) \gamma_c - 4 (bnsX + \gamma_c) \gamma_d]^2 - ns\xi} \tag{41}$$

and

$$\pi_c^* = \frac{a^2X(bnsX + \gamma_c) (bX + 2\gamma_d)^2}{[bX (bn (-n(1-s) - 2)) sX - 2bX (n(1-s) + 1) \gamma_c - 4 (bnsX + \gamma_c) \gamma_d]^2 - (1-s)n\psi} \tag{42}$$

respectively. These expressions show that, at the Nash equilibrium, the profits of defectors (cooperators) are positive provided that the applied sanctions (the costs to apply sanctions) are not too heavy. For example, under the assumption $\psi = 0$ and $\xi > 0$, we have π_c^* always positive and π_d^* positive or negative depending on the sanctions applied and on the number of cooperators.

We now relax the assumptions of a fixed fish biomass X and of a fixed share s between cooperators and defectors. For the fish stock at time period t , denoted by $X(t)$, a logistic growth function in discrete time is assumed:

$$X(t + 1) = F(X(t)) = X(t) (1 + \alpha - \beta X(t)) - H^*(X(t), s) \quad (43)$$

Moreover, an evolutionary mechanism is introduced to describe how, at each time period, the dynamics of $s(t)$ is updated. For this purpose, the replicator dynamics in discrete time is employed, given by

$$s(t + 1) = s(t) \frac{\pi_c^*(t)}{\bar{\pi}} \quad (44)$$

where

$$\bar{\pi}(t) = s\pi_c^*(t) + (1 - s)\pi_d^*(t) \quad (45)$$

represents the average profit observed at time t (see e.g. Hofbauer and Sigmund 1998, 2003; Weibull 1995).

Of course, (44) states that $s(t + 1)$ will be greater than $s(t)$ if $\pi_c^*(t) > \bar{\pi}(t)$ whereas $s(t)$ will decrease if $\pi_c^*(t) < \bar{\pi}(t)$. As $\pi_c^*(t) > \bar{\pi}(t)$ if and only if $\pi_c^*(t) > \pi_d^*(t)$, it follows that the population share related to the better performing strategy at time period t increases in the next period.

Both $\pi_c^*(t)$ and $\bar{\pi}(t)$ depend on $s(t)$, as well as $X(t)$, so the difference Eqs. (43) and (44) define a two-dimensional discrete dynamical system in the dynamic variables $X(t)$ and $s(t)$, obtained by the iteration of a map of the plane $T : (X(t), s(t)) \rightarrow (X(t + 1), s(t + 1))$ given by

$$T : \begin{cases} X(t + 1) = X(t) (1 + \alpha - \beta X(t)) - H^*(X(t), s(t)) \\ s(t + 1) = s(t) \frac{\pi_c^*(X(t), s(t))}{s\pi_c^*(X(t), s(t)) + (1-s)\pi_d^*(X(t), s(t))} \end{cases} \quad (46)$$

where $H^*(X, s)$ is given by (39), with x_d^* and x_c^* defined in (37) and (38) respectively, and π_d^* , π_c^* , are given in (41) and (42) respectively.

It is straightforward to see that if $\pi_c^*(t) > 0$ and $\pi_d^*(t) > 0$ then $s(t) \in (0, 1)$ implying $s(t + 1) \in (0, 1)$ as well. However, negative profits may arise if the parameters ξ and/or ψ are positive. As the influence of these parameters is always related to the difference $\xi - \psi$, without loss of generality we shall assume $\psi = 0$, and whenever $\pi_d^*(t) < 0$ we shall assume $s(t + 1) = 1$ (instead of the meaningless $s(t + 1) > 1$, as obtained by simply applying (46)).

It is important to notice that if $s(t) = 0$ then $s(t + 1) = 0$ for each $t \geq 0$, and if $s(t) = 1$ then $s(t + 1) = 1$ for each $t \geq 0$, i.e. the two boundary lines $s = 0$ and $s = 1$ are trapping lines, on which the dynamics are governed by one-dimensional unimodal maps, given by the restrictions of the two-dimensional map (46) to them. These two cases correspond to particular benchmark cases, where

we have all cooperators and all defectors respectively, i.e. the cases considered in Szidarovszky and Okuguchi (1998, 2000) respectively, see also Bischi et al. (2005) on this point. The properties of these one-dimensional dynamical systems can be easily obtained. For example, the dynamics along the invariant edge $s = 0$, where all players are defectors, are governed by the one-dimensional map (43) with

$$H^*(X, 0) = nx_d^*(X, 0) = \frac{naX}{b(n+1)X + 2\gamma_d} \quad (748)$$

So the steady states are $X = 0$ and the solutions (if any) of the equation

$$b\beta(n+1)X^2 + (2\beta\gamma_d - \alpha b(n+1))X + na - 2\alpha\gamma_d = 0 \quad (47) \quad (749)$$

On the other invariant edge $s = 1$, where all players are cooperators, the dynamics are governed by (43) with

$$H^*(X, 1) = nx_c^*(X, 1) = \frac{naX}{2(bnX + \gamma_c)} \quad (752)$$

So the fixed points are $X = 0$ and the solutions (if any) of the equation

$$2b\beta nX^2 + 2(\beta\gamma_c - \alpha bn)X + na - 2\alpha\gamma_c = 0 \quad (48) \quad (753)$$

The analysis is the same as in Szidarovszky and Okuguchi (2000) or in Bischi et al. (2005).

As usual, the starting point for the qualitative analysis of a nonlinear dynamical system is the localization of the steady states and the study of their local stability. The steady states of the two-dimensional dynamical system (46) are the fixed points of the map T , solutions of the system $T(X, s) = (X, s)$. It is straightforward to see that two *corner equilibria* always exist, given by $E_0 = (0, 0)$ and $E_1 = (0, 1)$, characterized by extinction of the resource. Other boundary equilibria may exist along the invariant lines $s = 0$ and $s = 1$, given by the solutions, if any, of Eqs. (47) and (48) respectively. If two equilibria with positive fish stock exist both on the invariant edge $s = 0$ and on invariant edge $s = 1$, say $X_2(0)$, $X_1(0)$ and $X_2(1)$, $X_1(1)$ respectively, then, on the basis on the arguments of Sect. 3, the following relation must hold: $X_2(1) < X_2(0) < X_1(0) < X_1(1)$. A necessary condition for the existence of two positive equilibria along $s = 0$ is that two positive equilibria exist along $s = 1$. However, it may happen that two positive equilibria exist along $s = 1$ and no positive equilibria exist along $s = 0$.

The stability of these equilibria with respect to the one-dimensional dynamics trapped inside the invariant edges can be easily deduced from the discussion on the one-dimensional dynamics. However, the stability with respect to perturbations transverse to the invariant edges, i.e. what happens if a few defectors appear starting from a situation with all cooperators, or what happens if a few cooperators appear starting from a situation with all defectors, is more interesting: are such small

mutations eliminated by the evolutionary dynamics, so that the original benchmark case is restored (case of transverse stability) or do these behaviors grow up thus causing an irreversible departure from the original benchmark case?

An answer to these questions requires the study of the local stability of the boundary equilibria, i.e. the localization, in the complex plane, of the eigenvalues of the Jacobian matrix of (46) computed at the boundary steady states. This is not difficult in principle, as eigenvalues are always real because the Jacobian matrix of (46) is a triangular matrix along the invariant edges. This implies that we can only have nodes or saddle equilibria on the boundaries. However, the expressions of the eigenvalues are quite involved, and the obtained stability conditions are not easily interpreted.

Interior equilibria, i.e. steady states characterized by coexistence of cooperators and defectors, are obtained solving the nonlinear system

$$\begin{cases} X(\alpha - \beta X) = H^*(X, s) \\ \pi_c^*(X, s) = \pi_d^*(X, s) \end{cases} \quad (49)$$

with $0 < s < 1$. The set of points of the plane (X, s) that satisfy the first equation represent locus of points that give one-period stationary resource stock, i.e. $X(t + 1) = X(t)$. This set of points may be formed by two branches, say $X_1(s)$ and $X_2(s)$, with $X_2(s) < X_1(s)$ for each s . Moreover, $X_2(s)$ is a decreasing function and $X_1(s)$ is increasing, so the branch $X_1(s)$ has positive slope and the branch $X_2(s)$ has negative slope in the plane (X, s) . The intersection of the branch $X_1(s)$ ($X_2(s)$) with the invariant edge $s = 1$ gives the X coordinate of the boundary equilibrium $X_1(1)$ ($X_2(1)$), and the same holds for the intersections (if any) with the other invariant edge $s = 0$. However, it may happen that the two branches intersect $s = 1$ but have no intersections with $s = 0$, because they may merge for $s > 0$. The knowledge of these curves give us the following information: starting from a given point (X, s) , a one-step iteration of (46) generates a new point $(X', s') = T$, rank-1 image of (X, s) by T , with $X' > X$ if (X, s) is in the strip between the curves $X_2(s)$ and $X_1(s)$ (or between the axis $X = 0$ and the curve $X_1(s)$ if only the branch $X_1(s)$ exists) and with $X' < X$ if (X, s) is on the left of the curves $X_2(s)$ (provided it exists) or on the right of the curve $X_1(s)$.

A similar reasoning can be applied to the set of points that satisfy the second Eq. (49), which represents the locus of points that give one-period stationary population share, i.e. $s(t + 1) = s(t)$. A numerical solution of the equation $\pi_c^*(X, s) = \pi_d^*(X, s)$ for different sets of parameters gives rise to decreasing curves in the plane (X, s) . The points above the curve $\pi_c^*(X, s) = \pi_d^*(X, s)$ are points where $\pi_c^*(X, s) < \pi_d^*(X, s)$, hence the rank-1 images $(X', s') = T(X, s)$ of points (X, s) above the curve have $s' < s$. Of course, the points below the curve are characterized by $\pi_c^*(X, s) > \pi_d^*(X, s)$, hence $(X', s') = T(X, s)$ are such that $s' > s$. These arguments allow us to obtain a global qualitative picture of the dynamic behavior of the dynamical system (46).

Any intersection between the curve $\pi_c^*(X, s) = \pi_d^*(X, s)$ and one of the curves $X_i(s)$, $i = 1, 2$, represents a steady state of the dynamical system. The steady states along the branch $X_2(s)$ cannot be stable, because all the points of that branch behave as repelling points along the X direction. Instead, steady states located along the branch $X_1(s)$ are candidates to be stable equilibria.

The results given in Bischi et al. (2004) can be summarized as follows. First, the presence of many cooperators always leads to a relatively high level of the resource stock, hence wealth in the long run, whereas many defectors can cause a severe depletion of the resource, incrementing also the basin of attraction of steady states with extinction of the resource.⁴ Second, from the point of view of a regulator, not only the cooperative behavior can be supported by increasing the level of sanctions, or lowering the prices, but also it is possible to reach steady states characterized by the presence of only cooperators. These equilibria may be transversely stable, i.e. even if a defector enters the market, this behavior does not spread over the population. Otherwise, low sanctions and/or high prices could lead to steady states with only defectors, and the potential depletion (even extinction) of the resource. In this case, even if the system starts from an initial condition with only cooperators, the non-cooperative behavior will prevail if only one agent decides to defect.

5 Conclusions

In this paper, we have described in details three evolutionary models of commercial harvesting of renewable resources. In particular, we have addressed the presence of heterogeneity in the fish populations to be harvested, in the rules established in different aquatic regions or in the behaviors of exploiters, more or less oriented towards cooperation. In most cases, these assumptions are motivated by real cases or by authorities' proposals for harvesting policies to regulate fisheries in the Italian Adriatic Sea. In any case, the underlying economic structure, the spirit of the analysis and the guiding principles are certainly gained from Prof. Okuguchi's inspiring works on commercial fisheries.

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⁴See Bischi and Lamantia (2005) for the topological definition and for an overview on nonlinear dynamical models in discrete time.

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