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A note on the paper 'On dynamical multi-team Cournot game in exploitation of a renewable resource'

Gian Italo Bischi [a,*], Michael Kopel [b], Ferenc Szidarovszky [c]

[a] DESP – Department of Economics, Society, Politics, University of Urbino, Italy
[b] Department of Organization and Economics of Institutions, University of Graz, Austria
[c] Department of Applied Mathematics, University of Pecs, Hungary

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ABSTRACT

In a recent paper (Asker, 2007) [1] a dynamic Cournot oligopoly game is proposed and it is claimed that this model represents competition among firms that exploit a common access natural resource. According to the author's claim, the feature that relates the model with renewable natural resource harvesting is given by the presence of a particular cost function where the total cost of each fisherman is proportional to the square of the own quantity of harvesting and inversely proportional to the total harvesting quantity. In contrast, the usual function used in the literature on the exploitation of natural resources (such as fisheries) is inversely proportional to the available resource stock, and not to the total harvesting. This, in some sense, assumes exactly the opposite (as the available resource is inversely proportional to the total harvesting). So, we believe that the paper (Asker, 2007) [1] contains an error which is probably due to a misunderstanding or a misreading and misinterpretation of the (well-established) literature on bioeconomic modelling, but nevertheless misleading to researchers interested in bioeconomic modelling. The aim of this short note is to explain the mistake and to summarize the correct derivation and interpretation of the cost function. Our goal is to avoid the propagation of a subtle (but nevertheless misleading) error.

1. Why the cost function is wrong

Let $X(t)$ denote the quantity of a renewable resource (e.g. total fish biomass in the case of fisheries) available at time t and let $q_i(t)$ be the quantity harvested by firm i, with $i = 1, \dots, N$. Hence, $Q_T(t) = \sum_{i=1}^{N} q_i(t)$ is the total quantity harvested by all firms at time t.

The relation between the market price and the total quantity harvested is given by an inverse demand function (or "price function") $p(t) = f(Q_T(t))$. For example, in [1] a linear function is proposed, $p(t) = a - bQ_T(t)$. A cost function widely used in the literature on harvesting of renewable resources has the form

$$C_i(t) = c_i + \gamma_i \frac{q_i^2(t)}{X(t)}, \tag{1}$$

where c_i represents a fixed cost and $\gamma_i > 0$ a positive coefficient. This cost function essentially captures the fact that it is more costly to harvest a scarce resource. That is, the cost of harvesting increases if X decreases. A derivation of such a particular type of cost function can be found in several classical books and papers. Indeed, following [4,6,5,2,3], let us assume that current harvesting q_i is obtained through a Cobb-Douglas production function of the resource stock X and harvesting effort E_i exerted by firm i (see e.g. [4], pages 222–223.) with total factor productivity ρ

* Corresponding author. Tel.: +39 0722330434.
E-mail addresses: gian.bischi@uniurb.it (G.I. Bischi), michael.kopel@uni-graz.at (M. Kopel), szidarka@gmail.com (F. Szidarovszky).

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$$q_i(X, E_i) = \rho X^{\alpha} E_i^{\beta}$$

from which $E_i = \rho^{-1/\beta} X^{-\alpha/\beta} q_i^{1/\beta}$. Moreover, assuming that the "production function" $q_i(X, E_i)$ is an homogeneous function of degree 1 with $\alpha = \beta = \frac{1}{2}$ and that the cost of fishing is proportional to exerted effort, i.e. $C_i = \delta_i E_i$, we get

$$C_i = \delta_i \rho^{-2} X^{-1} q_i^2 = \gamma_i \frac{q_i^2}{X}.$$

In contrast, the cost function proposed in [1] is given by

$$C_i(t) = c_i + \frac{q_i^2(t)}{Q_T(t)} \tag{2}$$

thus stating that the harvesting cost decreases if total harvest is increased and consequently if the resource is more scarce. This, however, does not make any (economic) sense and seems to be due to an evident misunderstanding. In fact, an increase in the total harvesting quantity $Q_T(t)$ leads to a decrease in available resource biomass according to the typical dynamic equation that regulates the time evolution of a renewable natural resource stock,

$$\frac{dX}{dt} = G(X) - Q_T,$$

in continuous-time or

$$X(t+1) = X(t) + G(X(t)) - Q_T(t)$$

if discrete time is assumed, where $G(X)$ represents a growth function (e.g. a logistic function or something similar). In any case, it is evident that $X(t)$ and $Q_T(t)$ are inversely related. As a consequence, the cost function (2) proposed in [1] is quite different from the correct form of the harvesting cost function (1) and hence misleading for other researchers not familiar with the literature on mathematical modelling of renewable resources.

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