5 The Role of Competition, Expectations and Harvesting Costs in Commercial Fishing

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1 Introduction

In this chapter oligopoly theory and population dynamics are combined to describe the exploitation of a living resource under imperfect competition. This requires an interdisciplinary approach, because profit maximization arguments must be combined with the biological laws which regulate the natural growth of living resources in order to determine the long-run behavior of the natural system. The results obtained are often characterized by complex behavior and bifurcation phenomena, because the interaction of human economic decision making with ecological dynamics are highly non linear (see e.g. Rosser Jr., 2001). Indeed, resource economics is a very important field of application of dynamic analysis. In the literature on the economics of renewable resources, a plethora of questions has been studied using the tools from optimal control theory, dynamic programming and the theory of nonlinear dynamical systems. Renewable resources, e.g. grass, trees or fish, have the capacity for reproduction and growth over time and their stock is diminished by the harvesting activities of a sole owner or several individuals. Among the problems which have been investigated extensively are: How do (optimal) harvesting paths look like? Under which conditions is it more likely to observe conservation or extinction of the resource? What is the influence of the market structure? In the last 50 years considerable progress has been made, in particular, in fishery economics (for an overview, see Conrad 1995). To appreciate the breadth of topics which are the focus in the economics of fisheries and for a presentation of the main economic insights, the interested reader is advised to consult the book by Clarke (1990). Furthermore, the texts of Conrad and Clarke (1987) and Conrad (1999) can be recommended. In this chapter, we will introduce a bioeconomic model of commercial fishing to study the evolution of the stock of a fish population which is subject to harvesting over time. We will focus on three points: (i) the influence of the market structure, in particular, the role of strategic effects; (ii) the problem of extinction or conservation of the resource and the role of harvesting costs; (iii) the influence of errors in predicting the fish stock when determining future harvesting activities.

In order to address the first problem, we will analyze the model dynamics under imperfect competition in a duopoly framework and compare it with the dynamics under the assumption that the rights to harvest the resource are held by a sole owner. Equivalently, we can interpret this as a situation where the competitors form a cooperative venture. In many papers on the dynamics of renewable resources it has been assumed that the sea is open access, i.e. the fish stock is harvested by a large number of unregulated, competitive fishermen with no barrier to entry or exit. Due to perfectly competitive markets for harvested fish, the price for fish has been taken to be constant. Here, however, we assume that due to some form of regulation, e.g. limited entry, access to the fishery is restricted. Furthermore, we assume that the resource is offered on two distinct markets with downward sloping demand. The issue of international trade has been introduced recently into commercial fishing models by Okuguchi (1998) and Szidarovszki and Okuguchi (1998). Following their terminology, we will refer to the two markets in our model as the home market and the foreign market.

With respect to problem (ii), we will try to provide some insights under which circumstances conservation of the resource is more likely to be observed and how this depends on the harvesting costs. The costs of harvesting are crucial since they have a direct impact on the (profit-maximizing) behavior of the fishermen. For example, a regulator can influence the costs of harvesting by such methods as restricting the length of the fishing season, setting total catch limitations, and regulating the type of fishing gear used (see Clark 1990). Furthermore, costs can be reduced by giving R&D subsidies (see Okuguchi 1998). This in turn determines the level of the (optimal) harvesting activities of the fishermen. Again, the duopoly case will be compared to the cooperative venture case to see the effects of harvesting costs and competition on conservation of the resource.

In dealing with (iii), we are studying the problem that fishermen are only boundedly rational. When they determine their optimal level of future harvesting activities, they do not know the future fish stock. Instead, all they have is an imperfect prediction, which is revised as new information becomes available. To be a little bit more precise, let $X(t)$ denote the biomass or number of individuals in a fish stock. Then the actual evolution of the fish stock over time in the absence of fishing is determined by a so-called growth function $G(X)$, which is often expressed as $G(X) = XR(X)$, where R is the specific (or unitary) growth rate. A widely used form for R is the logistic growth

$$R(X) = (\alpha - \beta X) . \tag{1}$$

The parameter α is referred to as the intrinsic growth rate and $K = \alpha/\beta$ is called the carrying capacity. If an extra mortality term due to harvesting $h(t)$ is included, the dynamics of the fish stock is governed by $X\,(t+1) - X(t) = X\,(\alpha - \beta X) - h(t)$. In resource economics it is usually assumed that fishermen, when they determine the level of harvesting $h(t)$, have precise knowledge of this relation. However, in the real world this will hardly be the case. More realistically, economic agents have access to a collection of past data about the harvested amounts of fish and some other indicators of the size of the fish population, from which they then try to derive an estimate of the future fish stock, $X^e(t)$. With every new piece of information, the estimate $X^e(t)$ will be updated and be used to determine the future harvesting activities. Obviously, there are many ways to model how agents derive an estimate from past data. We will assume a simple learning rule called adaptive expectations, which states that the new estimate is a weighted average of the previous estimate and current data about the actual fish stock, where the weight on the past estimate (or belief) is a measure of the inertia of the agents. This leads to a two-dimensional dynamical system, where the dynamic variables are the actual value and the expected value of the fish stock. The study of the global properties of this two-dimensional system gives us some interesting insights on the combinations of true and expected values of the fish stock which lead to survival or extinction in the long run. As we shall see, this information is obtained through the study of the basins of attraction, which reveals another source of complexity, related to the complex structure of the basins' boundaries. This can be studied by the method of critical curves (see Mira et al., 1996, or the chapter 3 of this book). In fact, the two dimensional map, whose iteration gives the time evolution of the duopoly model with adaptive expectations, is noninvertible. This feature causes the creation of complex topological structures of the basins, such as non-connected or multiply connected sets, even if the attracting sets are very simple, e.g. stable fixed points (on this topic see also Bischi and Kopel, 2001).

This chapter is organized as follows. First, we make some remarks on the logistic growth model for different values of the intrinsic growth rate, since this has been an important topic in the literature. We then investigate the dynamics of the fish stock under imperfect competition. We are considering the effects of harvesting costs and adaptive expectations on the possibility of conservation and compare the results with the case of a sole owner. We end the chapter with some concluding remarks.

2 The logistic growth model

To begin with, we summarize some results on the dynamics of the unharvested fish population. As mentioned above, in the absence of any harvesting, the stock of the fish population in period t is determined by the discrete time logistic equation

$$X(t+1) = X\left(1+\alpha-\beta X\right). \tag{2}$$

The dynamical behavior of this equation has been studied extensively by May (1976, 1987) and May and Oster (1976). See also Conrad and Clark (1987).

The map in (2) is conjugate to the standard logistic map $z' = \mu z(1-z)$ with parameter $\mu = 1+\alpha$ through the linear transformation $X = (1+\alpha)z/\beta$. For any $\alpha > 0$ there are two fixed points

$$X_0^* = 0 \quad \text{and} \quad X_1^* = \frac{\alpha}{\beta}. \tag{3}$$

The first represents a particular biological equilibrium, known as extinction of the species, the second is called "carrying capacity" of the species when no harvesting occurs. The equilibrium point $X_0^* = 0$ is unstable for each $\alpha > 0$, and the positive equilibrium X_1^* is stable for $0 < \alpha < 2$. For $2 < \alpha < 3$, even if X_1^* is unstable, a bounded positive attractor exists around it, characterized by oscillatory dynamics (periodic or chaotic) and trapped inside the absorbing interval $I = \left[(1+\alpha)^2\left(3+2\alpha-\alpha^2\right)/16\beta, (1+\alpha)^2/4\beta\right]$. For each $0 < \alpha < 3$, the basin of attraction of the positive attractor is the interval

$$\mathcal{B} = \left(0, \frac{1+\alpha}{\beta}\right).$$

Note that any initial condition out of this interval would generate a trajectory with negative values, i.e. it leads to extinction of the fish population in finite time (see e.g. Clarke, 1990, p.13).

3 A Duopoly Model

In order to study the three questions stated in the introduction, namely the interplay between harvesting costs and extinction, the impact of competitive forces and the influence of adaptive expectations, we consider the following model of international commercial fishing. Two countries (the duopolists or players) harvest fish and sell it in their home market and in the foreign market. The inverse demand functions for the markets $i = 1, 2$ are given by $p_i = a_i - b_i(x_{1i} + x_{2i})$, where $x_{ki}(t)$ denotes the amount of fish harvested by player $k = 1, 2$ and sold in market i at time period t. Each player's harvesting costs depend on the harvest rate and, additionally, on the total fish stock. This latter assumption captures the fact that it is easier and less expensive to catch fish, if the fish population is large. Let $X(t)$ be the total fish biomass at time t in the common sea and $h_k(t) = x_{k1}(t) + x_{k2}(t)$ the amount of fish *harvested* by player k at time t. Then the cost function of player k is given by $C_k = c_k + \gamma_k h_k^2/X$, which satisfies the common assumptions that costs are convex in the fish stock and concave in harvest (see Clark 1990). Note that players might be heterogeneous with respect to their costs. In such a case the effect of cost leadership of one player on the resulting equilibrium can be investigated. Let $s_i(t) = x_{1i}(t) + x_{2i}(t)$ be the amount of fish *supplied (and sold)* in country i at time period t. We assume that the total fish harvested by the two competitors equals the total fish supplied in the two markets, i.e. $H(t) = h_1(t) + h_2(t) = s_1(t) + s_2(t)$.

3.1 Reaction functions and Nash equilibrium

Let $X^{e_j}(t)$ denote player j's expectation at time $t-1$ of the fish stock prevailing in the sea at time t. Furthermore, let $x_{ki}^{e_j}(t), j \neq k$ be player j's expectation at time $t-1$ of the amount of fish offered for sale by rival k in market i at time t. Then, the expected profits are

$$\pi_1^e(t) = \left[a_1 - b_1(x_{11} + x_{21}^{e_1})\right] x_{11} + \left[a_2 - b_2(x_{12} + x_{22}^{e_1})\right] x_{12} - c_1 - \gamma_1 \frac{h_1^2(t)}{X^{e_1}(t)}$$

$$\pi_2^e(t) = \left[a_1 - b_1(x_{11}^{e_2} + x_{21})\right] x_{21} + \left[a_2 - b_2(x_{12}^{e_2} + x_{22})\right] x_{22} - c_2 - \gamma_2 \frac{h_2^2(t)}{X^{e_2}(t)}$$

We assume that players are only boundedly rational. They try to determine their harvesting activities such that their current expected profit is maximized. The first order conditions for firm 1 are:

$$\frac{\partial \pi_1^e}{\partial x_{11}} = a_1 - 2b_1 x_{11} - b_1 x_{21}^{e_1} - 2\gamma_1 \frac{h_1(t)}{X^{e_1}(t)} = 0$$
$$\frac{\partial \pi_1^e}{\partial x_{12}} = a_2 - 2b_2 x_{12} - b_2 x_{22}^{e_1} - 2\gamma_1 \frac{h_1(t)}{X^{e_1}(t)} = 0$$

from which

$$x_{11} = \frac{a_1}{b_1} - (x_{11} + x_{21}^{e_1}) - 2\frac{\gamma_1}{b_1}\frac{h_1}{X^{e_1}}$$
$$x_{12} = \frac{a_2}{b_2} - (x_{12} + x_{22}^{e_1}) - 2\frac{\gamma_1}{b_2}\frac{h_1}{X^{e_1}}$$

follows. Accordingly, the optimal harvesting quantities for player 1, x_{1i}, for the markets $i = 1, 2$ depend on the predictions player 1 makes of the quantities offered by its rival and, additionally, on the expected fish stock. Equivalently, from the first order conditions for player 2 we get that the optimal quantities of player 2 depend on player 2's predictions of the quantities offered by its rival, $x_{11}^{e_2}$ and $x_{12}^{e_2}$, and the expected fish stock X^{e_2}.

For simplicity, we assume that the duopolists have homogeneous expectations and that the players are aware of this, i.e. $X^{e_1}(t) = X^{e_2}(t) = X^e(t)$. Furthermore, since we are interested in the equilibrium harvesting quantities, we assume that $x_{ki}^{e_j} = x_{ki}, i, j, k = 1, 2; j \neq k$. Given these assumptions, solving the system above would yield the optimal quantities of fish, $x_{ki}^*, i, k = 1, 2$, harvested by player k and sold in country i as a function of the (current) expectation X^e. Here instead, we focus on the total amount of harvest by player k. Adding the equations above yields

$$h_1 = 2A - (h_1 + h_2) - 2\frac{B\gamma_1}{X^e} h_1$$

where $A = 0.5\,(a_1/b_1 + a_2/b_2)$ and $B = (1/b_1 + 1/b_2)$. The parameter A is the average of the market volumes of the two markets. The parameter B is a measure of the (price-)sensitivity of demand. If prices in both markets drop by 1 unit, then total demand for fish increases by B units. From this equation, we can get the reaction function for player 1,

$$h_1 = \frac{A}{1 + \frac{B\gamma_1}{X^e}} - \frac{1}{2(1 + \frac{B\gamma_1}{X^e})} h_2.$$

Analogously, we get the reaction function for player 2,

$$h_2 = \frac{A}{1 + \frac{B\gamma_2}{X^e}} - \frac{1}{2(1 + \frac{B\gamma_2}{X^e})} h_1.$$

These two functions, represented by straight lines, are depicted in fig. 1. Obviously, in the duopoly case the harvesting quantities are strategic substitutes, i.e. if one player increases its quantity offered on the market, the other player's optimal reaction is to reduce its quantity.

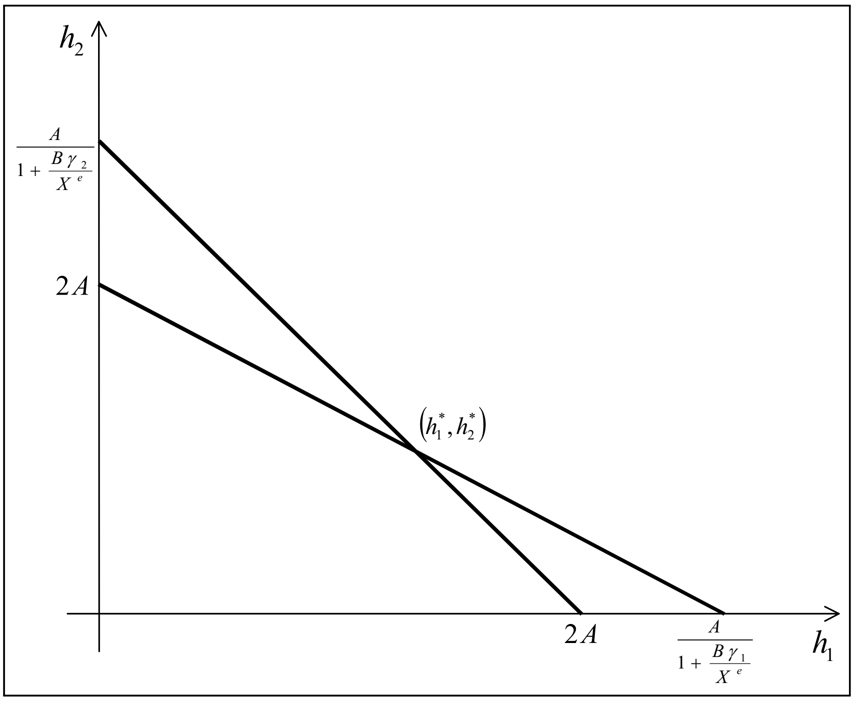

Figure 1: *Reaction curves*

The intersection point of the two lines defines the Nash equilibrium. From the graphical representation several properties of the Nash equilibrium quantities can be easily derived. For example, if the market volume A increases, then both reaction curves shift to the right. Accordingly, both Nash equilibrium harvesting quantities increase. Furthermore, strategic effects can also be studied. If the cost parameter γ_2 increases, then the intersection point of the reaction curve of player 2 with the vertical axis moves downwards. As a consequence, the harvesting quantity of player 2 in equilibrium (optimally) decreases. This is due to the direct cost effect. At the same time, player 1 optimally increases the harvesting quantity, which is a consequence of a strategic effect. The harvesting quantities of the players in the Nash equilibrium can be calculated as

$$h_1^* = 2A\frac{1+\frac{2B\gamma_2}{X^e}}{4(1+\frac{B\gamma_1}{X^e})(1+\frac{B\gamma_2}{X^e})-1}$$
$$h_2^* = 2A\frac{1+\frac{2B\gamma_1}{X^e}}{4(1+\frac{B\gamma_1}{X^e})(1+\frac{B\gamma_2}{X^e})-1}.$$

By defining

$$f(x) = \frac{1}{1+\frac{2B\gamma_1}{x}} + \frac{1}{1+\frac{2B\gamma_2}{x}}.$$

we can rewrite these expressions as

$$h_1^* = \frac{2A}{(1+\frac{2B\gamma_1}{X^e})(1+f(X^e))} \tag{4}$$
$$h_2^* = \frac{2A}{(1+\frac{2B\gamma_2}{X^e})(1+f(X^e))}.$$

Hence, the total amount of harvesting of the two players is

$$H^*(t) = h_1(t) + h_2(t) = 2A\frac{f(X^e(t))}{1+f(X^e(t))} = \tag{5}$$
$$= 4A\frac{(1+\frac{B(\gamma_1+\gamma_2)}{X^e})}{4(1+\frac{B\gamma_1}{X^e})(1+\frac{B\gamma_2}{X^e})-1}.$$

The optimal harvesting quantities of player j in (4) depend on the average of the market volumes (A), on the sensitivity measure of demand (B), and on player j's cost parameter γ_j. Note that the equilibrium activities of the two players also depend on the player's expectation of the fish stock (X^e). This effect is usually absent in quantity-setting duopoly models and, hence, worth to be investigated in more detail. An increase in the (common) expectation of the fish stock affects both players. One might be tempted to argue that they both extend their harvesting activities. However, in a situation where the total amount of harvesting offered in the market is already very high, doing so would decrease the prices on the markets and would lower profits. Accordingly, it seems reasonable that the relation between (marginal) harvesting costs play a crucial role in determining the optimal reaction of the players. Calculating the derivative $\partial h_1^*/\partial X^e$, one notices that the sign depends on the sign of the expression

$$(2\gamma_1 - \gamma_2)X^2 + 4B\gamma_1\gamma_2 X + 4B^2\gamma_1\gamma_2^2.$$

Hence, for $\gamma_2 < 2\gamma_1$ we can be sure that the derivative is positive. On the other hand, for $\gamma_2 > 2\gamma_1$ it turns out that the derivative is positive only if the expected fish stock is sufficiently small (otherwise negative). An analysis of the derivative $\partial h_2^*/\partial X^e$ yields the same qualitative results with the indices of firm 1 and 2 exchanged, i.e. for $\gamma_2 > 0.5\gamma_1$ the derivative is positive; for $\gamma_2 < 0.5\gamma_1$ it is positive only if the expected stock is sufficiently small. Summing up, our analysis reveals that if the cost parameters of the duopolists do not differ too much, i.e. $0.5\gamma_1 < \gamma_2 < 2\gamma_1$, both players increase their harvesting activities if they predict a higher fish stock. Surprisingly, however, if the cost advantage of one firm is considerable ($\gamma_2 < 0.5\gamma_1$ or $\gamma_1 < 0.5\gamma_2$), then the cost leader increases its quantity only for sufficiently small values of the expected fish stock. If the expected fish stock is sufficiently high, then the cost leader reduces its harvesting effort! The reason is that the cost leader's harvesting is already high due to the high expected fish stock and due to its low marginal cost. In such a situation it is optimal, even if a higher fish stock is predicted, to reduce harvesting.

3.2 Perfect foresight for both countries

Taking into account (5), the evolution of the fish stock subject to harvesting by the duopolists is governed by

$$X(t+1) = X(t)\left(1 + \alpha - \beta X(t)\right) - \frac{2Af(X^e(t))}{1 + f(X^e(t))} \tag{6}$$

where the parameters γ_i, α, β, A and B are positive. By specifying how the players form their common expectation X^e, we obtain a dynamic model. To have a benchmark available, we start from the case where both players have perfect foresight, i.e.

$$X^e(t) = X(t) \text{ for each } t.$$

This means that the players are able to accurately predict the fish biomass which will prevail in the sea in the next period or, in other words, that the players know the dynamic equation which governs the evolution of the fish population. The model (6) becomes

$$X(t+1) = F(X(t))$$

where

$$F(x) = x(1+\alpha-\beta x) - H(x) = x(1+\alpha-\beta x) - 2A\frac{f(x)}{1+f(x)}$$

with

$$f(x) = 2x\frac{x+B(\gamma_1+\gamma_2)}{(x+2B\gamma_1)(x+2B\gamma_2)}.$$

The fixed point $X_0^* = 0$ always exists, and positive fixed points, if any, are obtained as solutions of the equation

$$\alpha - \beta x = \frac{H(x)}{x}. \tag{7}$$

With respect to the one-dimensional map of the logistic growth function, which describes the unharvested population, the presence of harvesting $H(x)$ makes the unimodal curve lower and an inflection point may exist. This implies that the map may have three fixed points, say $X_0^* = 0 < X_1^* < X_2^*$, with X_0^* and X_2^* stable and X_1^* unstable.

If the players have equal cost parameters, $\gamma_1 = \gamma_2 = \gamma_D$, then total harvesting becomes $H(x) = 4Ax/(3x+2B\gamma_D)$ and the equation to find the positive fixed points becomes

$$3\beta x^2 + (2\beta\gamma_D B - 3\alpha)x + 2(2A - B\alpha\gamma_D) = 0$$

Hence, if the common cost parameter is sufficiently large, namely

$$\gamma_D > \frac{2A}{B\alpha}, \tag{8}$$

the unique positive fixed point

$$X_{2,D}^* = \frac{3\alpha - 2\beta\gamma_D B + \sqrt{(3\alpha+2\beta\gamma_D B)^2 - 48A\beta}}{6\beta} \tag{9}$$

exists. Two positive solutions of the quadratic equation exist if $\gamma_D < 2A/B\alpha$ and $48A\beta < (3\alpha + 2\beta\gamma_D B)^2$. No positive fixed points exist if $48A\beta > (3\alpha + 2\beta\gamma_D B)^2$.

Given the results on the local stability of the steady states in the case of homogeneous players, we can derive some preliminary conclusions concerning the conservation of the renewable resource. Obviously, conservation or extinction of the resource depends on the relative magnitude of the intrinsic growth rate α of the renewable resource in comparison with the market volume, the price sensitivity of the consumers, and the cost parameter γ_D. From a regulator's point of view, α and β are biologically given, and A and B are the given market conditions (of course, they could be influenced by the firms' marketing activities). Accordingly, only the parameter γ_D can be influenced by a regulator in order to achieve conservation of the fish population. If the value of the cost parameter is sufficiently high ($\gamma_D > 2A/B\alpha$), extinction of the resource can be prevented (see fig. 2a). Note that the lower the intrinsic growth rate or the larger the market volume, the higher the cost parameter has to be in order to achieve conservation. If $\gamma_D < 2A/B\alpha$, extinction can occur. However, the situation depends on the interplay between the biological and the market parameters. If two fixed points X_1^* and X_2^* exist (see figs. 2b and 2d), then conservation results only if the stock of the renewable resource is in an intermediate range (larger than X_1^*, but smaller than $X_{1(-1)}^*$). For smaller or larger sizes of the population, extinction occurs either asymptotically (see fig. 2b) or in finite time (see fig. 2d). Observe that if the fish stock is larger than $X_{1(-1)}^*$, then natural mortality due to overpopulation together with the harvesting activities reduce the fish stock below the level X_1^*. Finally, if the parameter of the harvesting costs is sufficiently small, then no fixed point exists, and the resource is driven extinct independent of the fish stock (see fig. 2c).

Several remarks should be made concerning our results. First, it makes sense that extinction does not occur if costs are sufficiently high. This effect has been observed before in bioeconomic models of the fishery (see Neher 1990, Pearce and Turner 1990, Clark 1990). It might be surprising, however, that for $\gamma_D < 2A/B\alpha$ extinction occurs despite the fact that harvesting costs increase for a decreasing fish stock. The reason is simply that in this situation for small sizes of the fish stock the optimal harvest (which is about $2A/B\gamma_D$) is greater than the internal growth rate α of the resource. Finally, observe that a fish population can be viewed as a capital asset (see e.g. Dawid and Kopel, 1997, or Neher, 1990). The owners of the resource expect the asset to earn dividends at the normal rate of return; otherwise they would be tempted to dispose the asset. With respect to the fish stock, the owners compare the intrinsic growth rate of the resource with the rate of return of harvesting the resource and investing the proceeds. Extinction occurs, when the intrinsic growth rate is not sufficiently high. Looking at the local stability results, apparently the owners in our model behave according to this view (see figs. 2a-c). However, note that viewing the fish stock as a capital asset is an inherently intertemporal issue, i.e. the decision is made by considering the trade-off between harvesting now (and investing the returns) or let the resource grow (at the internal growth rate) and harvest in the future. The owners in our model are, in contrast to this, not forward-looking. They determine the harvest such that the expected profit of the current period is maximized. In doing this, they do not take into account the effects of their decision today on the fish stock and their opportunities tomorrow.

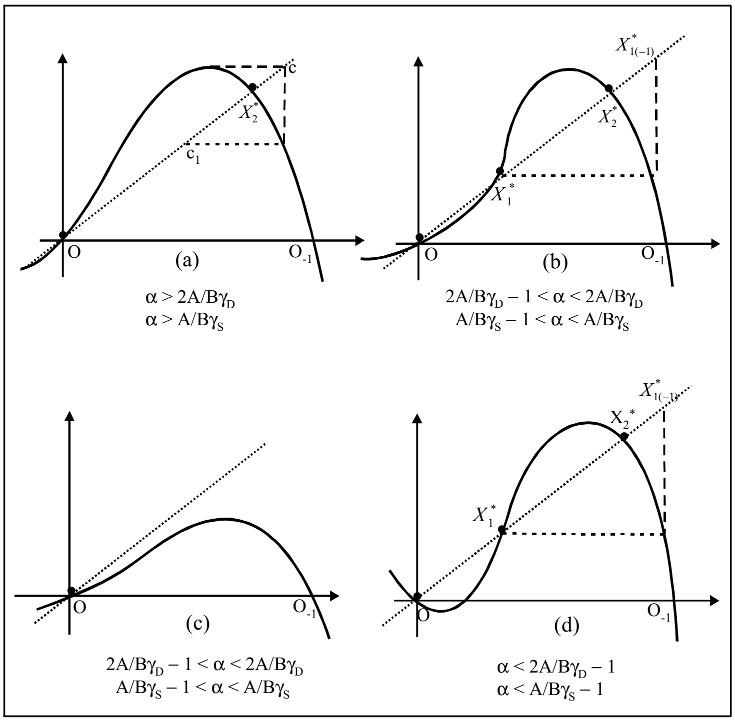

Figure 2: *One dimensional map F(x) whose iteration gives the dynamics with perfect foresight.*

If the players are heterogeneous with respect to their cost parameters, conditions for the existence of the positive fixed points are harder to come by. However, conditions for the stability of the origin can be easily found and a qualitative description of the resulting dynamics can be given. For $\alpha = A(\gamma_1 + \gamma_2)/B\gamma_1\gamma_2$, we have $F'(0) = 1$, and a transcritical bifurcation occurs. For

$$\alpha > A\frac{\gamma_1 + \gamma_2}{B\gamma_1\gamma_2},$$

we have $F'(0) > 1$. The fixed point X_0^* is unstable and only a positive fixed point X_2^* exists, where $X_2^* < \alpha/\beta$. The positive equilibrium may be stable or unstable and surrounded by a bounded attractor, as in the case of the unharvested resource. For

$$A\frac{\gamma_1 + \gamma_2}{B\gamma_1\gamma_2} - 1 < \alpha < A\frac{\gamma_1 + \gamma_2}{B\gamma_1\gamma_2}$$

we have $0 < F'(0) < 1$. The fixed point X_0^* is stable and two situations are possible. First, a pair of positive fixed points exists, $X_1^* < X_2^*$, unstable and stable, respectively. X_1^* is the boundary which separates the basin of X_0^* and that of X_2^*. Second, no positive fixed points exists and all the bounded trajectories converge to X_0^* (in finite or infinite time). For values of the intrinsic growth rate slightly smaller than $A(\gamma_1 + \gamma_2)/B\gamma_1\gamma_2$, we expect that the situation with three fixed points occurs. For $A(\gamma_1 + \gamma_2)/B\gamma_1\gamma_2 - 1 > \alpha$, there exists a neighborhood around X_0^* such that $F(X_0^*)$ is negative. This can be interpreted in saying that extinction is reached in finite time, as X_0^* is not an equilibrium in the biological sense, because the species goes extinct before reaching it (see also Clark 1990, p. 13). Observe that the set $\mathcal{B}$ of initial conditions which generate positive and bounded trajectories is given by $\mathcal{B} = (0, O_{-1})$,where $O_{-1} < (1 + \alpha)/\beta$ is obtained by solving the equation

$$1 + \alpha - \beta x = \frac{H(x)}{x}.$$

4 Comparison with the cooperative venture case

For the sake of comparison, let us assume that the two players agree to form a cooperative venture. Equivalently, we might think of this as a situation where the property rights for the resource are in the hands of a sole owner, which may be imagined either as a private firm or a government agency that owns complete rights to the exploitation of the fish population. The sole owner harvests fish and sells it not only in its home market, but also in a foreign market. The inverse demand functions for the markets $i = 1, 2$ are given by $p_i = a_i - b_i x_i$, where $x_i(t)$ denotes the amount of fish harvested and sold in market i at time period t. Let $X(t)$ be the total fish biomass at time t in the common sea and $h(t) = x_1(t) + x_2(t)$ the amount of fish *harvested* (and sold) at time t. Then the cost function is given by $C = c + \gamma_S h^2/X$. Furthermore, let $X^e(t)$ denote the owners prediction at time $t-1$ of the fish stock prevailing in the sea at time t. Then, the expected profit in period t reads

$$\pi^e(t) = [a_1 - b_1 x_1]\, x_1 + [a_2 - b_2 x_2]\, x_2 - c - \gamma_S \frac{h^2(t)}{X^e(t)}.$$

If we assume that the sole owner is only boundedly rational in the sense that he determines the harvesting activities such that the expected profit of the subsequent period is maximized, it is easy to see that the optimal harvesting quantity is

$$h^*(t) = \frac{A}{1 + \frac{B\gamma_S}{X^e(t)}} \tag{10}$$

where A and B are defined as before. The optimal harvesting quantity of a sole owner can be compared with the total harvesting quantity of the duopolists. If we assume that the cost parameters of both players are equal, i.e. $\gamma_1 = \gamma_2 = \gamma_D$, then the total amount of harvesting in equilibrium in the duopoly case is $H^*(X^e) = 4A/(3 + 2B\gamma_D/X^e)$, according to (4). It is easy to see that the following holds: If $\gamma_S \geq 0.5\gamma_D$, then for each level of X^e, $H^*(X^e) > h^*(X^e)$. In other words, competition has the effect that the resource is exploited more heavily for each level of expected fish stock. This has been noticed before in the literature, for example, in connection with open-access exploitation (see Clark 1990), and also in the context of a differential game model of fishing (see Levhari and Mirman 1982, and also Dutta and Sundaram 1993). Note, however, that this result does not give any information on the relation between the harvesting activities in the long run, i.e. in the long run steady state.

If we put the logistic growth function and the expression (10) for optimal harvesting together, then the dynamics of the fish stock subject to harvesting by a sole owner is determined by

$$X(t+1) = X(t)\left(1+\alpha-\beta X(t)\right) - \frac{A}{1+\frac{B\gamma_S}{X^e(t)}}. \tag{11}$$

If we assume that the owner can accurately predict the level of the fish stock in period t, i.e. $X^e(t) = X(t) \;\; \forall t$, we obtain a one-dimensional dynamical system. The time evolution of the fish population is then described by the iteration of the map

$$X' = F(X) = X\left(1+\alpha-\beta X\right) - \frac{AX}{X+B\gamma_S}. \tag{12}$$

The steady states of this dynamical equation are the non negative fixed points of the map (12). They are given by $X_0^* = 0$ and the positive solutions of the equation

$$\beta X^2 - \left(\alpha - \beta\gamma_S B\right)X + A - \alpha\gamma_S B = 0.$$

If

$$\alpha > \frac{A}{\gamma_S B}, \tag{13}$$

holds, then the slope at X_0^* is $DF(0) = 1+\alpha - A/\gamma_S B > 1$. Hence, X_0^* is unstable and there is a unique positive equilibrium given by

$$X_{2,S}^* = \frac{\alpha - \beta\gamma_S B + \sqrt{\left(\alpha+\beta\gamma_S B\right)^2 - 4A\beta}}{2\beta} \tag{14}$$

which may be stable or unstable (with an attractor around it). The equilibrium lies inside the absorbing interval $[c_1, c]$ (like in the situation shown in fig. 2a) with basin of attraction $\left(O, O_{(-1)}\right)$. If

$$\alpha < \frac{A}{\gamma_S B} < \alpha + 1$$

then $0 < DF(0) < 1$, and no positive equilibria exist if $A > \left(\alpha+\beta\gamma_S B\right)^2/4\beta$. On the other hand, two positive equilibria $0 < X_1^* < X_2^*$ exist, where X_2^* is given by (14), if $A < \left(\alpha+\beta\gamma_S B\right)^2/4\beta$ and $\beta\gamma_S B < \alpha$. In this case, X_1^* is unstable. The other equilibrium, X_2^*, is stable with basin given by

$\mathcal{B} = \left(X_1^*, X_{1(-1)}^*\right)$, where $X_{1(-1)}^*$ is the rank-1 preimage of X_1^* (compare fig. 2b). When X_2^* is unstable, then $\mathcal{B}$ is the basin of the absorbing interval $[c_1, c]$ around X_2^*, provided that $c < X_{1(-1)}^*$. Finally, if

$$\alpha < \frac{A}{\gamma_S B} - 1, \tag{15}$$

then we have a right neighborhood of $X_0^* = 0$ where $F(X)$ is negative.

We can now make a comparison between the stability conditions for the duopoly case and the case of a sole owner in order to see the effects of harvesting costs and competition on the conservation of the fish population. First, note that the threshold (8) for the duopolists's common cost parameter γ_D resulting in conservation of the resource independent of the fish stock is twice as large as for the case of a sole owner, see (13). If we consider, for example, the existence and stability of the fixed points for $A = 5.5, B = 2, \beta = 1, \alpha = 3$, then conservation of the resource independent of the fish stock is achieved if $\gamma_1 = \gamma_2 = \gamma_D > 1.833$, whereas in the case of a sole owner, the value of the cost parameter only needs to be half as large. If we consider, for example, $\gamma_S = 1$, a sole owner with perfect foresight would achieve conservation of the resource, whereas competition between only two players leads to extinction of the resource. Interesting insights can be gained if we look at the long run evolution of the fish population in the two cases. Let us assume that $\gamma_S = 1$ and $\gamma_D = 2$. Recall, that in such a situation, for each level of X^e, $H^*(X^e) > h^*(X^e)$. In other words, a sole owner would harvest a smaller quantity than the duopolists together. However, for the particular set of parameters we are considering, in the steady states $X_{2,S}^*$ and $X_{2,D}^*$ the relation between the harvesting quantities is reversed: The sole owner achieves a higher permanent catch, $H^*(X_{2,D}^*) = 2 < h^*(X_{2,S}^*) = 2.24$, where $X_{2,S}^* = 1.37 > X_{2,D}^* = 1$. So, competition between the duopolists may lead to overconsumption, with less left for future periods.

With respect to conservation, in the case of homogeneous costs, it is easy to see that a more general statement can be made. If n firms harvest the resource, then their common cost parameter must be n times as large in order to achieve conservation of the resource, i.e. $\gamma_O > nA/B\alpha$ (the subscript 'O' stands for oligopoly). Hence, this is another indication that competition between players might have a detrimental effect on the resource, unless the costs of the rivals are considerably higher than the corresponding costs of a sole owner. This is of particular interest if we discuss the effect of market entry (see also Szidarovszki and Okuguchi 1998). Imagine a situation in which a sole owner harvests fish and offers it on the two markets. Due to economies of scale and learning effects, the sole owner's value of the cost parameter is quite low, e.g. $\gamma_S = 1$. Despite such low costs, no matter what the actual fish stock, conservation of the resource results. In this situation a potential entrant is considering market entry. If entry is accommodated by the incumbent (the sole owner), then we are dealing with a duopoly situation just analyzed. The remarkable effect is that even if the new firm has considerably higher costs, e.g. $\gamma_2 = 5$, extinction will result.

In the heterogeneous case, the cost parameters of *both* players play a crucial role, as the following simple example demonstrates. Let us assume that $A = 1, B = 1, \beta = 1, \alpha = 2$. Now consider the situation of a sole owner and assume that the value of the cost parameter is $\gamma_S = 1$. Since $\gamma_S > A/B\alpha = 0.5$, it follows that the fish population survives independent of the initial fish stock. Consider, on the other hand, a duopoly situation where the players are heterogeneous with respect to costs. Suppose player 1 has a value of the cost parameter of $\gamma_1 = 1$. Then $A(\gamma_1 + \gamma_2)/B\gamma_1\gamma_2 = 1 + 1/\gamma_2$, and conservation of the resource depends on the value of the cost parameter of player 2. If player 1 is the cost leader, i.e. $\gamma_2 > \gamma_1 = 1$, then $1 + 1/\gamma_2 < \alpha = 2$, and the resource is preserved no matter what the initial fish stock is. However, if player 2 is the cost leader, then conservation of the fish stock depends on the initial stock of fish. If player 2's cost advantage is considerable (γ_2 less than about 0.23), then extinction occurs for all initial stocks of the resource. The higher optimal harvesting activity by player results, in this case, in extinction of the resource no matter what the initial stock of fish is!

5 Duopoly and Adaptive expectations

We now drop the assumption that the players can precisely predict the evolution of the fish stock. Instead, we assume that the duopolists form their common beliefs by using the adaptive expectations scheme. The model describing the evolution of the actual and the expected fish stock becomes two-dimensional:

$$T : \begin{cases} X(t+1) = X(t)\left(1 + \alpha - \beta X(t)\right) - 2A\frac{f(X^e(t))}{1+f(X^e(t))} \\ X^e(t+1) = (1-\lambda)\, X^e(t) + \lambda X(t) \end{cases} \tag{16}$$

where

$$f(x) = 2x\frac{x + B\left(\gamma_1 + \gamma_2\right)}{\left(x + 2B\gamma_1\right)\left(x + 2B\gamma_2\right)}.$$

Hence, for each t the duopolists have homogeneous expectations, where $\lambda \in (0, 1]$ denotes the inertia in revising expectations. A higher value of λ coincides with a higher willingness to take new information into account. Note that the limiting case $\lambda = 1$ means that the competitors have naive expectations. The time evolution of realized and expected values is obtained by the iteration of the two-dimensional map $T : (X(t), X^e(t)) \rightarrow (X(t+1), X^e(t+1))$. On the horizontal axis of the state space the actual (or realized) values of the fish stock are measured, whereas along the vertical axis the expected values are measured at each time period. Hence, points in a neighborhood of the diagonal $X = X^e$ represent good estimates of the fish stock, whereas the points in the region above (below) the diagonal represent situations where the fish stock is over- (under-) estimated. From the second component of (16) it follows that the fixed points must be located along the diagonal $X^e = X$. This implies that in equilibrium the expectations coincide with the actual size of the fish stock. In other words, in equilibrium expectations are fulfilled and are, hence, rational. Accordingly, we have "Rational Expectations Equilibria" (REE), whose coordinates, obtained from the first equation in (16) with $X^e = X$, are the same as for the model with perfect foresight, namely the fixed point $O = (0, 0)$ and those obtained by solving equation (7).

For the local stability of the REE, we now have to consider the two-dimensional Jacobian matrix

$$DT(X, X^e) = \begin{bmatrix} 1 + \alpha - 2\beta X & -2A\frac{f'(X^e)}{[1+f'(X^e)]^2} \\ \lambda & 1 - \lambda \end{bmatrix}$$

computed at the fixed points. For example, for the equilibrium $O = (0, 0)$, the range of stability is

$$\alpha < A\frac{\gamma_1 + \gamma_2}{B\gamma_1\gamma_2} < 1 - \alpha\frac{1 - \lambda}{\lambda}$$

and, hence, smaller with respect to the case of perfect foresight ($\alpha < A(\gamma_1 + \gamma_2)/B\gamma_1\gamma_2$). This set is non-empty only if $\alpha < 1 - \alpha(1 - \lambda)/\lambda$, i.e.

$$\alpha < \lambda.$$

Of course, if $\alpha \geq 1$ then $\alpha < \lambda$ is never satisfied, and O is always unstable. This result, which is based only on a local stability argument, seems to suggest that extinction is less probable with adaptive expectations than with perfect foresight. However, in the two-dimensional adaptive expectation case, extinction might even occur if the fixed point $(0, 0)$ is unstable. Looking at (16), we realize that as long as the fish stock and the expected size of the fish population are positive, the expectation remains positive. On the other hand, due to natural mortality and harvesting, the fish stock may become negative. Even if $(0, 0)$ is a fixed point, i.e. an equilibrium of the dynamical system, it might not be a biological equilibrium, because trajectories starting close to it take on negative values for the fish stock. Such a situation has to be interpreted as extinction in finite time (see Clark (1990)). Only a rigorous study of the basins of the equilibria give further information if and when this occurs.

For the study of the basins it is important to notice, first, that the map (16) is a noninvertible map, because given $(X(t+1), X^e(t+1))$ several distinct preimages can be obtained by solving (16) with respect to $(X(t), X^e(t))$. As the map T is continuously differentiable, it is easy to obtain the equation of LC_{-1}, since it is included in the set of points at which the determinant of the Jacobian vanishes, i.e.

$$\det DT\,(X, X^e) = (1-\lambda)\,(1+\alpha-2\beta X) + \lambda H'(X^e) = 0. \qquad (17)$$

It is also easy to obtain the image of LC_{-1} by applying T, i.e. $LC = T\,(LC_{-1})$. These sets constitute the so called critical curves, which separate the phase plane into regions Z_k whose points have k preimages, or, equivalently, where k distinct inverses of T are defined (see e.g. Mira et al., 1996, or chapter 3 of this book). It is interesting to note that for $\lambda < 1$ the curve LC_{-1} can be expressed as

$$X = \frac{1}{2\beta}\left[1+\alpha+\frac{\lambda}{1-\lambda}H'(X^e)\right],$$

whereas if $\lambda = 1$ (i.e. in the case of naive expectations), then $\det DT$ never vanishes, since $\det DT = H'(X^e) = 2Af'(X^e)/\left[1+f(X^e)\right]^2 > 0$ for each X^e. So, for $\lambda = 1$ no critical curves exist. As we shall see in the following, the presence of critical curves may have important consequences on the structure of the basins' boundaries and on the occurrence of global bifurcations which cause qualitative changes in the structure.

We have seen that in a situation where the duopolists have perfect foresight, as harvesting becomes cheaper, the danger of extinction of the resource grows. A similar result seems plausible for the model (16) with adaptive expectations. In the remainder of this section, we give a more rigorous study of the relation between harvesting costs and the dynamics of the resource if fishermen have adaptive expectations. We also compare the results with the benchmark case of perfect foresight. In particular, as before, we are interested in the effects of changes in harvesting costs on the probability of conservation (or extinction). Again, we assume that the players are homogeneous with respect to their cost parameters, i.e. $\gamma_1 = \gamma_2 = \gamma_D$ and, in order to show some numerical simulations, we consider the following set of parameter values: $\alpha = 3$, $\beta = 1$, $A = 5.5$, $B = 2$. Recall that, in the case of a sole owner with perfect foresight, for $\gamma_S = 1$ conservation of the resource occurs independent of the initial size of the fish stock. In the duopoly case, conservation of the resource is achieved if player's cost parameters are such that $\gamma_D > 1.833$, i.e. the value of the cost parameter has to be at least 83.3% higher. Accordingly, if we assume $\gamma_D = 2$, then in the duopoly case under perfect foresight the resource is conserved no matter what the initial size of the fish population is. Moreover, the results on the existence of two positive fixed points X_1^* and X_2^* show that as long as $\gamma_D > 1.812$, conservation is achieved at least from some initial values of the resource stock (to be more precise, for initial stock sizes in the interval $[X_1^*, X_{1(-1)}^*]$). How does the situation change when players have adaptive expectations? The first thing to notice is that for $\gamma_D = 2$, extinction of the resource occurs for all initial values of the actual and the predicted fish stock. Hence, errors in the prediction of the fish stock lead to extinction unless costs are higher than in a situation with perfect foresight. For adaptive expectations with $\lambda = 0.3$, we show the basin of the positive equilibrium (X_2^*, X_2^*) for $\gamma_D = 4$ (fig. 3a), $\gamma_D = 2.6$ (fig. 3b), $\gamma_D = 2.575$ (fig. 3c), $\gamma_D = 2.573$ (fig. 3d) and $\gamma_D = 2.56$ (fig. 3e). Observe that the basin of the equilibrium (X_2^*, X_2^*), which for this set of parameters is the only stable steady state, is represented by the white regions, whereas the grey points denote the set of initial conditions leading to extinction. Also note that the horizontal axis represents the actual level of the fish stock, whereas the vertical axis represents the expected fish stock. Obviously, conservation of the resource depends both on the initial sizes of the fish stock and the expected fish stock. For example, for a high value of the cost parameter, if the player's prediction of the fish stock is roughly accurate (i.e. the initial condition is taken from a neighborhood of the diagonal) then conservation results except in the cases where the (the actual and the predicted) resource stock is quite high (see fig. 3a). On the other hand, if the players initially overestimate the size of the fish stock (i.e. the initial condition is chosen above the diagonal, with $X^e(0) \gg X(0)$), then this might result in extinction of the resource. Roughly speaking, the evolution is as follows. Since the player's expectation of the fish stock is high, harvesting effort is high. This, subsequently, results in a reduction of the actual fish population. Since λ is small, the prediction of the future fish stock remains high, and this leads to a further reduction of the fish stock. This, taken together with the small reproduction of the resource due to its small size, leads to extinction in finite time. Note, however, that the evolution of the trajectories might crucially depend on the initial values of stock and expectation. As an example, consider a situation where both the expected and the actual fish stock are small. Due to low harvesting activities, the fish stock increases and the expected value increases too. Eventually, the actual and the expected fish stock are rather large. Now, due to overpopulation and the increase in harvesting activities, the actual fish stock is severely diminished. The evolution of trajectories is about the same, no matter if extinction results or not. It now depends on the precise initial size of the fish population and the predicted stock if extinction occurs.

Observe that, given the information on the structure of the basins and the critical curves, we can make a qualitative prediction of the evolution of the trajectories, since each grey tongue depicted in figs. 3a-d is the preimage of the corresponding tongue to the right of it. This is due to the folding action of the noninvertible mapping T, see Mira et al. (1996), see also chapter 3 of this book. Indeed, the global bifurcations which change the topological structure of the basin of the positive equilibrium, from simply connected (figs. 3a-b) to multiply connected (fig. 3c) to the union of non connected portions (fig. 3e), can be easily explained on the basis of the theory of critical curves in terms of contacts between the basin boundaries and LC. For example, the creation of "grey holes" nested inside the white basin of the positive equilibrium are the preimages of the small portion of the grey region, indicated by an arrow in fig. 3c, which entered the region Z_3 after a contact between LC and the basin boundary. Analogously, the transition from a multiply connected into a non connected basin, i.e. from fig. 3c to fig. 3d, is a consequence of a contact between the basin boundary and LC near the point indicated by the arrow in fig. 3d. The splitting of white islands into pairs of smaller ones is due to another contact between LC and the basin boundary at the point indicated by the arrow of fig. 3e. As in the benchmark model, for lower values of the cost parameter, the probability of extinction increases in the case of adaptive expectations (see figs. 3b-d), i.e. the set of stock-expectation-combinations resulting in conservation of the resource shrinks.

Once harvesting costs reach a certain level, the situation changes rapidly and drastically. In order to demonstrate the effect of adaptive expectations

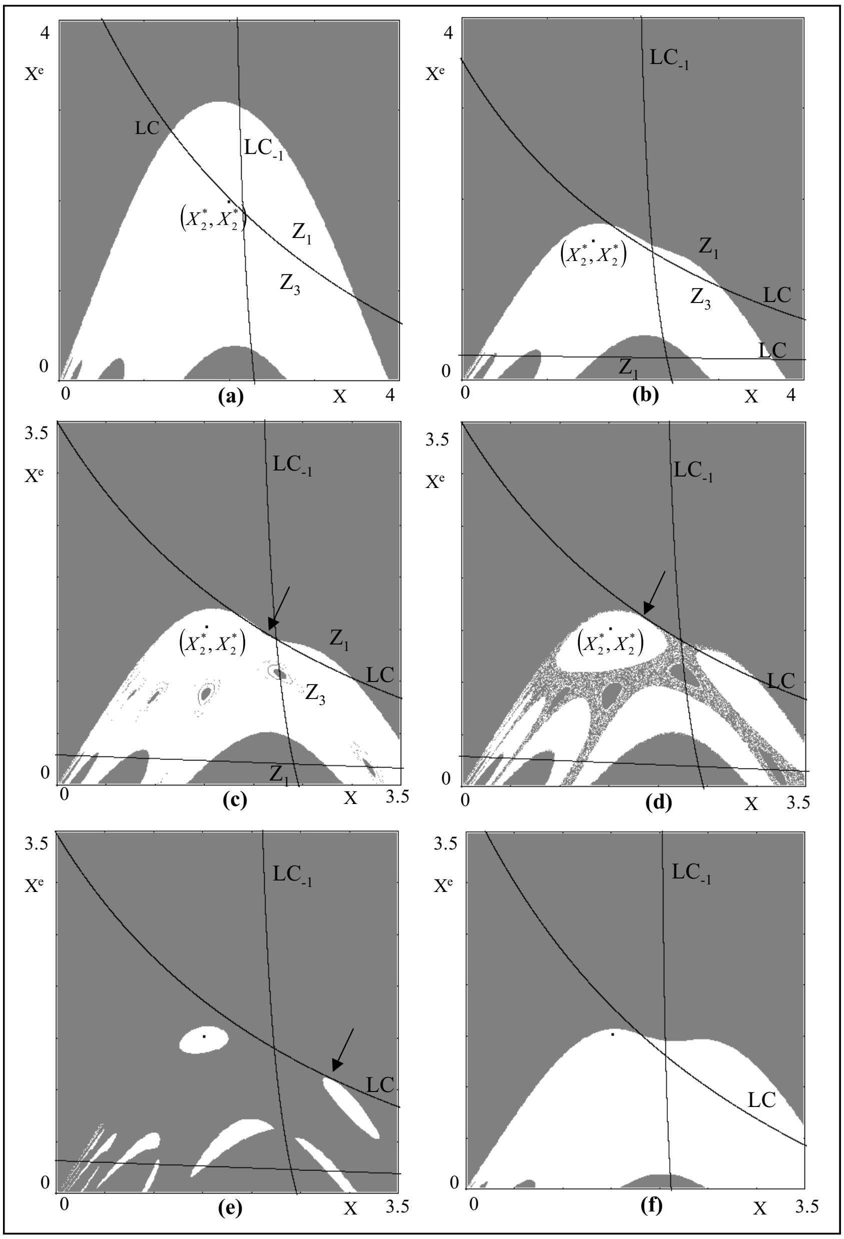

Figure 3: *Some numerical computations of the basins of the positive equilibrium (white regio). The grey region represents the set of initial conditions leading to extinction of the fish stock.*

with a different degree of inertia, we consider the same set of parameters as above, but with $\lambda = 0.1$ and $\gamma_D = 2.56$ (see fig. 3f). In this case, lower inertia in revising expectations enhance the stability of the positive equilibrium and, consequently, make extinction of the living resource less likely. However, it is worth noticing that the positive equilibrium is very close to the upper boundary of its basin. Despite the fact that the system is close to its equilibrium, a slight overestimation of the fish stock may lead to extinction of the fish population.

6 Concluding remarks

The combination of oligopoly games and resource economics is a challenging task, since the biological law which regulates the natural growth of the resource has to be taken into account. In fact, renewable resources, like a fish population, grow and decline over time. Due to their inherent dynamic nature, their evolution can only be understood using dynamic models. In particular, when the resource is subject to harvest, the problem of conservation is a complex issue. In this chapter we have proposed a discrete-time dynamic model which describes the decisions of duopolists engaged in commercial fishing, where each of the players sells the harvested fish both in a home and in a foreign market. Within this framework we have discussed several important topics, for example, the relation between harvesting costs and conservation of the resource, the influence of the market structure and the changes if agents are not able to predict the future evolution of the fish stock accurately. We studied the duopoly case in order to stress the role of competition and compared this situation with the case of a cooperative venture. Furthermore, adaptive expectations have been proposed to stress the role of expectations formation under bounded rationality, where we have confronted the results with the perfect foresight case.

The assumption of adaptive expectations has led us to a two-dimensional dynamical system in discrete time. Through a global analysis of the resulting noninvertible map, we have shown that, even if we limited our study to a set of parameters which give very simple attractors, namely stable fixed points, another source of complexity arises which is due to the creation of complex topological structures of the basins of attraction. Our analysis has been based on a combination of analytic, geometric and numerical methods. The global (or contact) bifurcations which causes the creation of complex structures of the basins have been studied by the method of critical curves (see Mira et al., 1996, Bischi and Kopel, 2001, see also the chapter 3 of this book).

Our main findings are quite intuitive and can be summarized as follows. Not surprisingly, higher harvesting costs tend to achieve conservation of the resource, independent of the fact if the right to harvest is held by a sole owner or if several players compete. Competition increases the probability of extinction. For example, if we take the case of a sole owner as a reference point and assume that harvesting costs of the sole owner are such that conservation is achieved, then with only two players, the harvesting costs of the competitors have to be twice as high to achieve the same result. Moreover, the numerical example given in this paper indicates that the long run fish stock in the duopoly case is smaller than in the cooperative venture case and the permanent catch in the latter situation is higher. If we, additionally (and realistically), assume that agents are not able to predict the future fish stock accurately and instead use an adaptive expectation scheme, extinction becomes more likely with respect to the case of perfect foresight, especially when fishermen overestimate the fish stock.

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