8. Macroeconomic fluctuations and heterogeneous agents

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1 INTRODUCTION

The *representative agent* assumption has been extensively adopted in economics either as the representative firm in the theory of production or as the representative household in consumption theory. This working hypothesis allows the analysis of microeconomic behaviour to be extended to macrorelationships in a straightforward way so that standard optimisation techniques can be used to explain aggregate data.

Recently, however, there have been signs of dissatisfaction with the representative agent framework. The game theoretic approach has shown that rational behaviour may generate multiple Nash equilibria. The asymmetric information approach has developed the idea that if agents differ from one another and we cannot discriminate among them, *adverse selection* can prevent a decentralised system from achieving even Pareto-constrained equilibria (Greenwald and Stiglitz, 1986). In particular, the growing New Keynesian literature often analyses 'real world' economies using a two-agents approach, in which small firms, usually financially constrained, have to interact with large firms who are not financially constrained (Bernanke et al., 1994).

Kirman (1992) and others have argued that the representative agent paradigm should be discarded in macroeconomics, and replaced with game theoretic models of economies consisting of strategically interacting agents. A less ambitious line of research replaces the representative agent assumption with a framework in which intrinsically different agents are distributed according to a density function but do not interact strategically. This chapter moves a modest step toward that approach by means of a study of macroeconomics based upon aggregation of groups of heterogeneous agents.

We build a model in two steps: in the first one (section 2) we model a maximising microeconomic behaviour, common to each firm of the universe; the second step (section 3) describes a model of ‘compartments’ which determines the distribution of the various firms among the classes, small and large, that is, the evolution of the population as influenced by the evolution of their equity base which determines their growth, entrance and exit from the market.[1] The two steps are linked by strategic substitutability which affects aggregate price level determination and the evolution of the equity base. Section 4 analyses the dynamical properties of the system. Section 5 concludes. Since the analytical tool developed in this model can be straightforwardly extended to more complicated models, we shall try to keep the analysis as simple as possible using, as a basic framework, the model put forward by Greenwald and Stiglitz (1993).

The main results of this exercise are: 1. fluctuations may depend on the distribution of the equity base, even if its level does not change; 2. fluctuations are asymmetric because of the death-birth process of firms; 3. large firms are less prone to fluctuations than small ones; 4. firm specific shocks spread over the whole economy; 5. processes are ergodic and time-dependent.

2 A SIMPLE MODEL OF FIRM’S BEHAVIOUR

In this section we build a simple model of firm’s behaviour when capital markets are imperfect, along the lines of Greenwald and Stiglitz (1993). The basic idea is that the choice of the scale of production is an intrinsically risky decision. In fact, inputs must be paid before output is sold so that a financing gap emerges. Due to asymmetric information, firms’ ability to raise funds by issuing new equities is limited: they must finance production by means of bank loans. Therefore, they run the risk of bankruptcy, which occurs whenever sales proceeds turn out to be lower than debt commitments.

We assume that each firm produces an undifferentiated good *(q)* by means of a one-to-one technology (uniform across firms) which uses only labour *(l)* as input[2]:

$$l = q \tag{8.1}$$

Aggregate output, Q_a, is

$$Q_a = q + Q \tag{8.2}$$

where Q is the volume of other firms' output. In order to simplify the argument, we will assume that the production of each firm is negligible if compared to the production of all the other firms, that is, $q<<Q$, so that total output 'almost' coincides with other firms' output: $Q_a \cong Q$. In the following, therefore, we will treat Q as aggregate output.

We assume that Q is not known to each firm. In other words, in the eyes of each firm, the total volume of output is a random variable. Moreover, we assume that there is a one-period time lag between the moment production is carried out and the moment transactions occur and sales proceeds are cashed in. Therefore the selling price of each firm 'tomorrow' will be a decreasing function of total output 'today':

$$p_{t+1} = a - bQ \tag{8.3}$$

where a and b are non-negative parameters, in particular we assume: $a > 1$; $0 < b < 1$. Of course, since total output is a random variable, also the selling price is stochastic.

The parameter a represents the maximum selling price: $a = p_{\max}$. From (8.3) it is clear that the market can absorb at most: $Q_{\max} = a / b$. $Q_{\max} > 1$.

We assume that Q has a uniform distribution over the interval $[0, a/b]$. As a consequence, also p_{t+1} has a uniform distribution over the interval $[0, a]$. In this case:

$$E(Q) = \frac{a}{2b} = \frac{1}{2} Q_{\max} \tag{8.3'}$$

$$E(p_{t+1}) = \frac{a}{2} = \frac{1}{2} p_{\max} \tag{8.3''}$$

Due to the time lag between production and sale, the firm must anticipate the wage bill. Firms can hire as much labour as they want at the nominal wage W. We define the financing gap of each firm in each period as the difference between the wage bill (Wq) and the equity base or net worth (A). Moreover, we assume that the financing gap can be filled only by means of bank loans. Firms can borrow at will on terms which must yield the lender a given nominal (gross) return of R.

Profit is equal to:

$$\Pi_{t+1} = p_{t+1}q_t - R(Wq_t - A_t)$$

and expected profit is:

$$E(\Pi_{t+1}) = E(p_{t+1})q_t - R(Wq_t - A_t) = RA_t + \left(\frac{a}{2} - RW\right)q_t$$

A sufficient condition for the non negativity of expected profit is:

$$\frac{a}{2} > RW \tag{A.1}$$

which means that the average production cost (interest payments per unit of output) is smaller than the expected price.

The firm runs the risk of bankruptcy, which occurs if total output is so high and the selling price of the firm is so low that revenue falls short of debt commitments, so that debt servicing becomes impossible. The bankruptcy condition, therefore, is:

$$p_{t+1}q_t < R(Wq_t - A_t) \tag{8.4}$$

or:

$$p_{t+1} < R\left(W - \frac{A}{q}\right) \equiv \underline{p} \tag{8.4'}$$

Substituting (8.3) into (8.4) and rearranging, we can rewrite the bankruptcy condition as follows:

$$Q > \frac{a}{b} - \frac{R}{b}\left(W - \frac{A}{q}\right) = Q_{\max} - \frac{R}{b}\left(W - \frac{A}{q}\right) \equiv \overline{Q} \tag{8.4''}$$

In other words, the firm goes bankrupt if competitors produce ‘too much’ (forcing the selling price to be ‘too low’), that is, their output is greater than a critical upper threshold $\overline{Q}$ (so that the selling price is smaller than a critical lower threshold $\underline{p}$). This volume of output in turn is a decreasing function of *W* and *R* and an increasing function of *A*. The lower *R* or *W* and the higher *A*, the higher is the volume of total output (the lower the selling price) that the firm can tolerate without going bankrupt.

In this framework, the probability of bankruptcy *(F)* is zero if net worth is so high that the firm can finance the wage bill by means of internal finance alone. In this case, in fact, the firm does not have to raise funds on the credit market and does not run the risk of bankruptcy:

$$F = 0 \quad \text{if} \quad Wq < A$$

In fact, if $Wq<A$, then $\underline{p} < 0$ and $\overline{Q} > Q_{\max}$: since the individual price cannot be negative and total output cannot be greater than the maximum size of the market, in this case bankruptcy is impossible.

On the other hand, the probability of bankruptcy is one if net worth is so low that the maximum volume of competitors' output that the firm can tolerate is non-positive. Since total output must be positive, in this case bankruptcy is a certain occurrence.

In symbols:

$$F = 1 \quad \text{if} \quad \overline{Q} \leq 0 \quad \therefore \quad q\left(W - \frac{a}{R}\right) \geq A$$

This condition implies also that the threshold price is higher than the maximum price. In fact:

$$\underline{p} > p_{\max} \quad \therefore \quad R\left(W - \frac{A}{q}\right) > a \;\Rightarrow\; q\left(W - \frac{a}{R}\right) \geq A$$

If the threshold price – that is, the lowest price that the firm can tolerate without going bankrupt - is higher than the maximum price, bankruptcy is a certain occurrence.

Notice that *F* is always smaller than one if the critical upper threshold $\overline{Q}$ is positive. This is true whenever $W - \frac{a}{R} < 0$ or $RW < a$, that is, the average production cost is smaller than the maximum price. If (A.1) holds true, the average production cost is smaller than the expected price, which is half the maximum price. Therefore the average production cost is smaller than the maximum price.

If $\left(W - \frac{a}{R}\right)q < A < Wq$, the probability of bankruptcy becomes:

$$F = \Pr(Q > \overline{Q}) = \left(Q_{\max} - \overline{Q}\right)\frac{b}{a} = 1 - \overline{Q}\,\frac{b}{a}$$
$$= 1 - \frac{\overline{Q}}{Q_{\max}} = \frac{R}{a}\left(W - \frac{A}{q}\right) = \frac{\underline{p}}{p_{\max}} \tag{8.5}$$

and $0 < F < 1$.

In figure 8.1 we represent the p.d.f. of total output. The distance OE represents maximum output. The vertical intercept of the p.d.f. is the reciprocal of this distance. The distance OD represents the highest volume of output that the firm can tolerate without going bankrupt. The probability of bankruptcy is the darkened area.

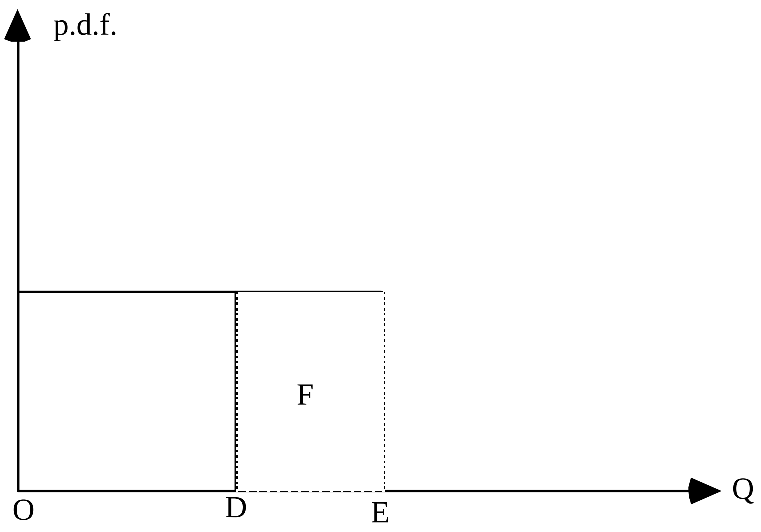


Figure 8.1

In figure 8.2 we represent the p.d.f. of the individual price. The distance OB represents maximum price. The vertical intercept of the p.d.f. is the reciprocal of this distance. The distance OA represents the lowest price that the firm can tolerate without going bankrupt. The probability of bankruptcy is the darkened area.

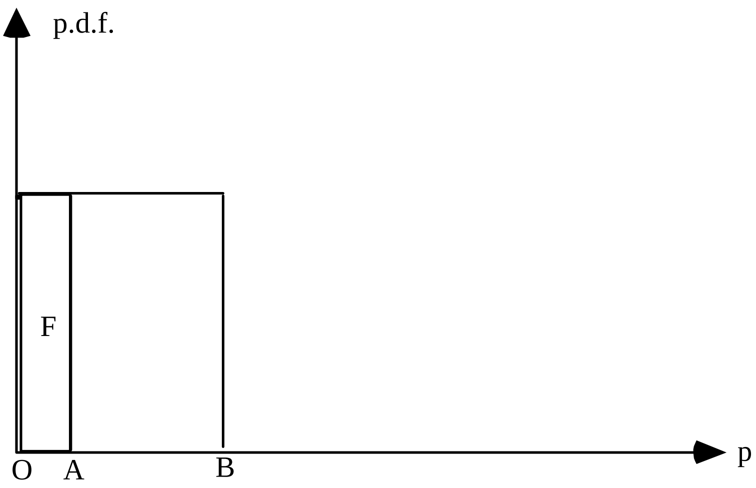


Figure 8.2

The probability of bankruptcy is the complement to one of the ratio of the highest volume of competitors' output that the firm can tolerate to maximum output. It is also equal to the ratio of the lowest price that the firm can tolerate to maximum price. Since $\overline{Q}$ is a decreasing function of *W* and *R* and an increasing function of *A*, the probability of bankruptcy is an increasing function of *W* and *R* and a decreasing function of *A*.

Figure 8.3 provides a simple geometric interpretation. Point $C \equiv (\overline{Q}, \underline{p})$ on the demand curve represents the lowest price and the highest output which the firm can tolerate. Point $B \equiv (0, p_{\max})$ represents the maximum price while point $E \equiv (Q_{\max}, 0)$ represents maximum output. Therefore, the probability of bankruptcy is:

$$F = \frac{\text{OA}}{\text{OB}} = \frac{\text{DE}}{\text{OE}} \cdot$$

An increase in RW or a decrease in A increases $\underline{p}$ and decreases $\overline{Q}$ so that point C moves along the demand curve towards point B and F increases.

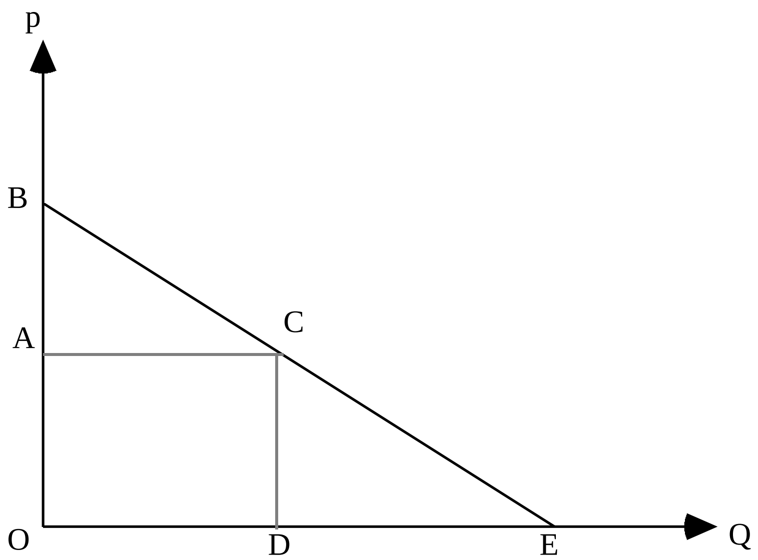


Figure 8.3

We assume that if the firm goes bankrupt, it incurs an additional (bankruptcy) cost which is an increasing (quadratic) function of the scale of activity:

$$\text{CB} = cq^2 \tag{8.6}$$

Notice that if the firm has an 'abundant' net worth (so that F=0), it has to maximise output in order to maximise (expected) profits. In this case the supply of each firm is:

$$q_t = \frac{A}{W} \tag{8.7}$$

If the financing gap is positive, that is, net worth is smaller than the wage bill, the firm maximises an objective function (say *V*) which is equal to expected profits less bankruptcy costs if bankruptcy occurs, that is, it chooses the level of output *q* by solving the following maximisation problem:

$$\underset{q}{\text{Max}}\, V = E(\Pi_{t+1}) - CB \bullet F = E(p_{t+1})q \; - R(Wq - A) - cq^2 F$$

Substituting (8.3") and (8.5) into the expression above we obtain:

$$\underset{q}{\text{Max}}\, RA_t + \left(\frac{a}{2} - RW\right)q \; - cq^2 \frac{R}{a}\left(W - \frac{A}{q}\right)$$

The FOC for a maximum of *V* yields the following supply function:

$$q_t = \bar{h} + h_a A_t \tag{8.8}$$

where $\bar{h} = \frac{a}{4c}\left(\frac{a}{RW} - 2\right)$, $h_a = \frac{1}{2W}$. Thanks to (A.1): $\bar{h} > 0$.

Finally we assume that there is a full capacity ceiling, that is: $q \leq \bar{q}$ where $\bar{q}$ is the maximum output that the firm can produce. All in all, we can write the individual supply function as follows:

$$q = H(A) = \begin{cases} \bar{h} + h_a A & \text{if } A < A' \\ \frac{1}{W} A & \text{if } A' \leq A \leq \bar{A} \\ \bar{q} & \text{if } A > \bar{A} \end{cases}$$

where $A = 2W\bar{h}$; $\bar{A} = W\bar{q}$; and $\bar{h} + h_a \bar{A} < \bar{q}$.

Figure 8.4a represents the individual supply function defined above.

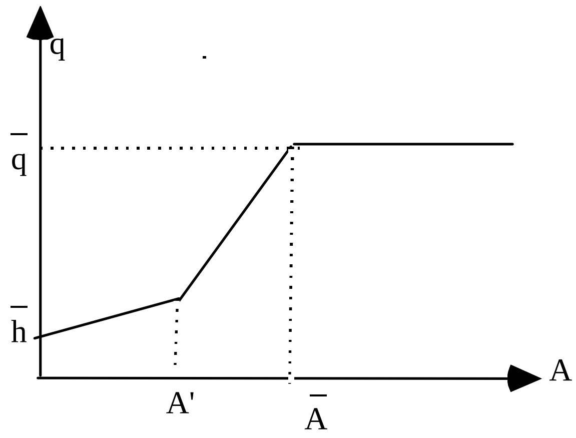


Figure 8.4a

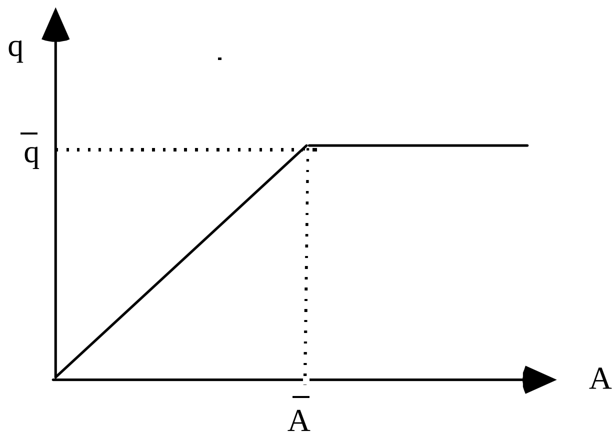


Figure 8.4b

If (A.1) does not hold true, the intercept of the straight line of equation (8.8) is negative, while the slope is smaller than that of equation (8.7). Therefore, in this case the individual supply function is

$$q = H(A) = \begin{cases} \frac{1}{W} A & \text{if } A \le \overline{A} \\ \overline{q} & \text{if } A > \overline{A} \end{cases}$$

Figure 8.4b represents the individual supply function in this case.

The equity base of each firm evolves according to the following law of motion:

$$A_{t+1} = p_{t+1} q_t - R(Wq_t - A_t) = p_{t+1} H(A_t) - R[WH(A_t) - A_t] \quad (8.9)$$

Net worth in period t+1 is equal to profits in the same period, which in turn depends on the realisation of the random variable represented by the individual price.

3 THE EVOLVING STRUCTURE OF THE CORPORATE SECTOR

Let's assume that there is a large number (N>>1) of firms which can be pooled into several classes depending upon their size, that is, the level of their net worth. We begin with the simplest case in which there are two classes of firms. Each firm can be labelled either *small* (class 1) - denoted by the suffix s - or *large* (class 2) - denoted by the suffix l. A small firm has an equity endowment ranging from A_0 to A_1 while a large firm has net worth of A_1 or higher.

$$class\,1 : I_s = (A_0, A_1),$$
$$class\,2 : I_l = (A_1, +\infty).$$

Notice that A_0 and A_1 represent two (arbitrarily chosen) levels of net worth which are used to identify the classes of firms. If net worth is smaller than A_0 the firm goes bankrupt.

For the sake of convenience, we will also define class 0 which represents a state of non-existence for firms. If firms come into existence, they leave class 0 to become part of class 1 or 2 according to their size. If firms go bankrupt they leave class 1 or 2 and reach class 0.

Let $x_{s,t}$ ($x_{l,t}$) be the number of small (large) firms at time t, and $A_{s,t}$ ($A_{l,t}$) the equity position of the representative firm for each class. Aggregate output al time t is,

$$Q_t = q_{s,t} x_{s,t} + q_{l,t} x_{l,t} \equiv H(A_{s,t}) x_{s,t} + H(A_{l,t}) x_{l,t}$$

while price at time t is a decreasing function of aggregate output at time t-1,

$$p_t = a - bQ_{t-1} = a - b[H(A_{s,t-1}) x_{s,t-1} + H(A_{l,t-1}) x_{l,t-1}]$$

Interclass dynamics can be represented as in figure 8.5.

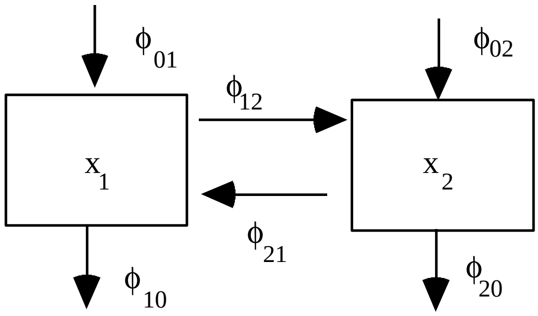


Figure 8.5

Figure 8.5 helps to identify six types of flows (states):

- ϕ_{01} (ϕ_{02}) is the flow of firms from class 0 to class 1 (2) – that is, the flow of new (just born) small (large) firms;
- ϕ_{10} (ϕ_{20}) represents the flow of firms from class 1 (2) to class 0 – that is, the flow of small (large) firms who go bankrupt (and die);
- ϕ_{12} (ϕ_{21}) is the flow of small (large) firms which become large (small).

We assume that:

(A.3) ϕ_{01} (ϕ_{02}) is proportional to the change of net worth that small (large) firms made in the previous period: $A_{s,t} - A_{s,t-1}$ $\left(A_{l,t} - A_{l,t-1}\right)$;

(A.4) ϕ_{10} (ϕ_{20}) occurs if the equity base of the small (large) firm is lower than the lower threshold $A_{s,t} < A_0$ $\left(A_{l,t} < A_0\right)$.

ϕ_{12} (ϕ_{21}) is proportional to $\lfloor A_{s,t} - A_1 \rfloor_+$ $\left(\lfloor A_1 - A_{l,t} \rfloor_+\right)$, where $[.]_+$ denotes the function 'positive part'. This means that the flow ϕ_{12} is activated if the equity base $A_{s,t}$ is greater than the threshold level A_1 (ϕ_{21} is activated if $A_{l,t} < A_1$).

Firms enter (leave) the market when there is an increase (decrease) of net worth, and change class whenever the threshold value is reached. Because of the non linear relationship between output and the equity base, the different classes of firms will respond differently to the same shock.[3] Moreover, if one leaves aside the representative agent hypothesis, it is possible to appreciate the role of each single type of sectoral shock in the overall composition of the economy. In fact, different and/or asymmetric effects of the same shock may arise if the equity base is different at different times.

The complete discrete-time dynamical system for the the two-class model is:

$$x_{s,t+1} = x_{s,t} + a_{01}\lfloor A_{s,t} - A_{s,t-1} \rfloor_+ - a_{10}\lfloor A_0 - A_{s,t} \rfloor_+ x_{s,t} - a_{12}\lfloor A_{s,t} - A_1 \rfloor_+ x_{s,t} + a_{21}\lfloor A_1 - A_{l,t} \rfloor_+ x_{l,t}$$

$$x_{l,t+1} = x_{l,t} + a_{02}\left[A_{l,t} - A_{l,t-1}\right]_+ - a_{20}\left[A_0 - A_{l,t}\right]_+ x_{l,t} + a_{12}\left[A_{s,t} - A_1\right]_+ x_{s,t} - a_{21}\left[A_1 - A_{l,t}\right]_+ x_{l,t}$$

$$A_{s,t+1} = p_{t+1} H_s\left(A_{s,t}\right) - R\left(W H_s\left(A_{s,t}\right) - A_{s,t}\right)$$

$$A_{l,t+1} = p_{t+1} H_l\left(A_{l,t}\right) - R\left(W H_l\left(A_{l,t}\right) - A_{l,t}\right)$$

$$p_{t+1} = \left\{a - b\left\lfloor H_s\left(A_{s,t}\right) x_{s,t} + H_l\left(A_{l,t}\right) x_{l,t} \right\rfloor\right\}_+$$

where

The system is rather complicated; therefore in the following section we simulate the behaviour emphasising the dynamical properties.

4 DYNAMIC BEHAVIOUR

In this section we simulate the dynamical behaviour of the two-class system, whose extension to an *n* class framework is straightforward. In particular, we test:

i) if the system is non-ergodic, that is, if exists path-dependence and, more generally, if different initial conditions yield different steady states;
ii) its response to a nominal shock, investigating for the presence of asymmetries in time series and real effects of a nominal shock;
iii) the presence of composition effects (co-existence of firms of different sizes) and their impact on the dynamics; finally,
iv) we compare the dynamical behaviour of our non-representative model and a traditional aggregative model with a representative agent (when our 2 classes boil down to 1).

Numerical simulations of the model show that, for a wide range of parameter values, the variables x_s, x_l, A_s, A_l (and consequently *p*) converge to stationary asymptotic values. In the following we show the results of some simulations obtained with parameter values a=20 and b=0.05 in the demand function, R=1.02, $\bar{q}_1 = 40, \bar{q}_2 = 50$, c=1, a_{01} =0.7, a_{02} =0.1, a_{10} =a_{20} =a_{21} =0.2 and with threshold values used to define bankruptcy and to distinguish small from large firms $A_0 = 0.5$ and $A_1 = 40$ respectively.

In figure 8.6, for each value of the nominal wage W in the range [9,21], we represent the asymptotic or long run values of the number of small and large firms (x_s and x_l) by means of thick and thin lines respectively.

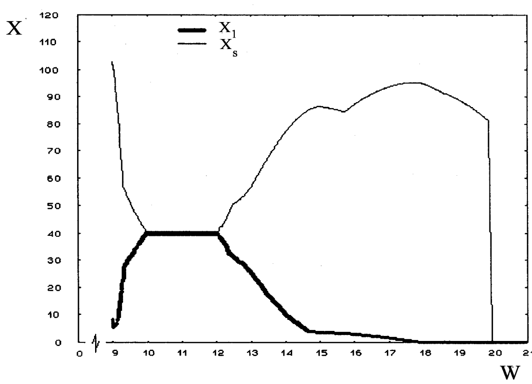
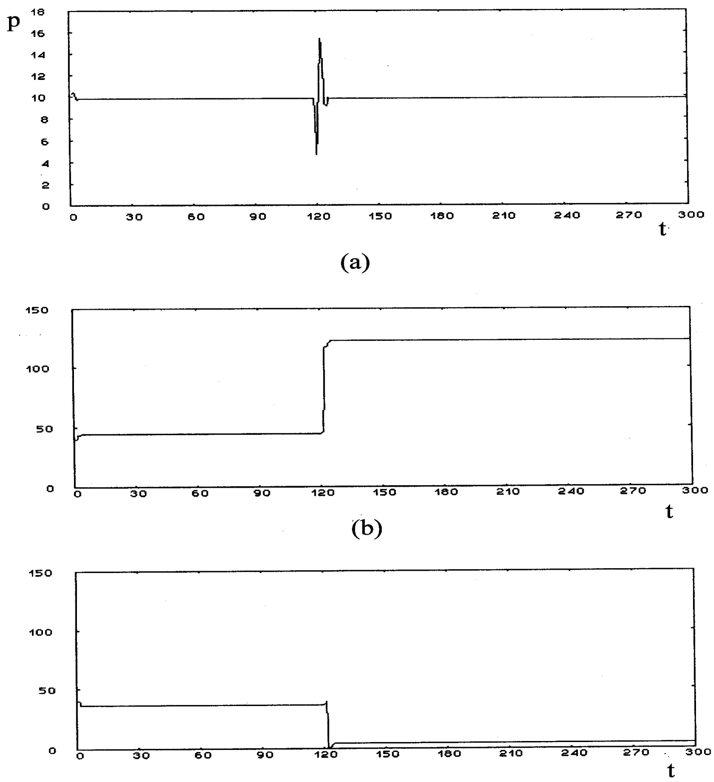


Figure 8.6

These values are obtained, for each W , starting from the initial condition $x_{s,0} = x_{l,0} = 40$ after a transitory run of 200 iterations has been discarded. It can be noticed that for $W \in (9{,}10)$, the initial configuration characterised by the same number of small and large firms leads to a long run equilibrium configuration in which small firms prevail, the difference $x_s - x_l$ being decreasing with *W*. In the range $W \in (10{,}12)$, the initial configuration characterised by the same number of small and large firms ($x_{s,0} = x_{l,0} = 40$) prevails also in the long run. For *W*>12, a long run equilibrium configuration is reached in which, once again, small firms prevail. Of course, for *W*>a, with a=20, both small and large firms undergo bankruptcy, therefore $x_s \to 0$ and $x_l \to 0$. However, the number of large firms decreases even before *W*>a : this is not due to bankruptcy, of course, but to the shrinking size of the large firms which become small ones. Moreover, it is worth noting that new small firms were born during the transitory dynamics because in the steady state the number of firms (both small and large) is greater than in the initial configuration. We recall that for *W > a/2 R =9.8,* assumption (A.1) does not hold true any longer, so that the supply function of figure 8.4a is replaced by that of figure 8.4b.

A peculiar feature of the model is the dependence of the asymptotic values of x_s and x_l on the initial condition $(x_{s,0}, x_{l,0})$. For example, when *W*=9.5, starting from the initial condition $x_{s,0} = 80,\ x_{l,0} = 40$ the asymptotic values of the number of firms in each class are $x_s = 170,\ x_l = 5$; from the initial condition $x_{s,0} = 20, x_{l,0} = 50$ the asymptotic values are $x_s = 110,\ x_l = 10$; finally, if the initial condition is $x_{s,0} = 20, x_{l,0} = 40$, both x_s and x_l remain constant over time.

We also investigate the effect of a nominal shock, that is, an exogenous change of the price level. First of all we compare the effect of that shock in our framework, characterised by two groups of firms (figure 8.7), with the case of an economy characterised by only one class of firms (the representative firm) (figure 8.8). The model with a representative firm is obtained just dropping a class from the general two-class model. In particular, it is characterised by $\bar{q} = 45$, $a_{01} = 0.4,\ a_{10} = 0.2$. Both figures 8.7 and 8.8 incorporate the hypothesis that *W*=9.5 (which means that assumption (A.1) holds true). With a representative firm, the shock is absorbed in a few periods and no real effects arise. On the other hand, with different classes of firms nominal *shocks are persistent and real effects arise*: There is in fact entry-exit from the market and changes of classes which destroy symmetry and produce real effects.



(a)

(b)

(c)

Figure 8.7

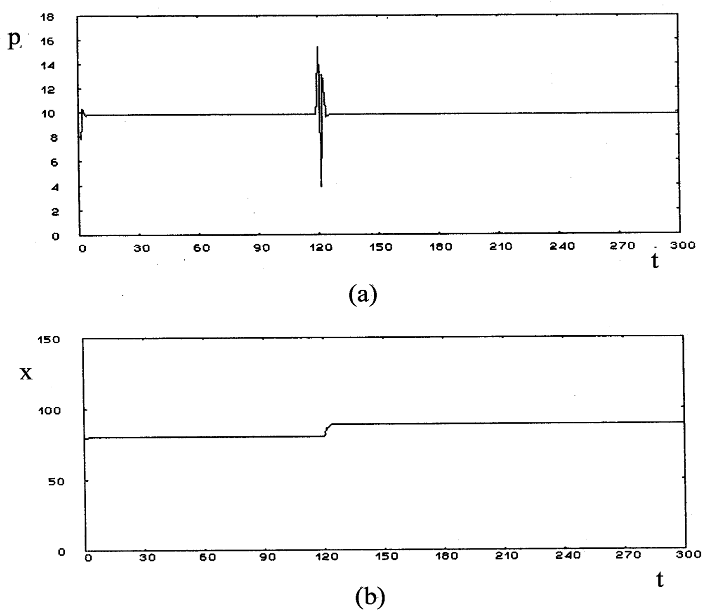


Figure 8.8

5 CONCLUDING REMARKS

To conclude this chapter we would like to draw the attention of the reader to the proper (which actually means very limited) use of economic tools like the representative agent hypothesis. The composition effect we have dealt with can be easily extended to several other cases. The main results of this exercise are: 1. fluctuations may depend on the distribution of the equity base, rather than its aggregate level; 2. fluctuations are asymmetric, since death and birth of firms have different timings; 3. nominal shocks have real effects.

Of course, this is only a first modest step toward the modelling of a macroeconomic model with heterogeneous agents.

NOTES

1. Mathematical models based on the subdivision of a system into compartments are widely used in physiology, pharmacokinetics and ecology (see e.g. Jacquez, 1972; Anderson, 1983; Godfrey, 1983).
2. In the following, non dated variables are referred to the current period.
3. Suppose two groups of firms with the same equity base operating at full capacity are subject to different sectoral shocks: a positive shock affects the equity base of group 1, while a negative shock of the same intensity affects the equity base of group 2. By construction, the average equity base is unchanged but average output decreases. Thus economic fluctuations can be traced back to sectoral adjustments, which normally do not appear when using the representative agent framework and whose effect could not be appreciated because, if the equity base is unchanged, total output will also be unchanged.

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