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Gaining the competitive edge using internal and external spillovers: a dynamic analysis

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Abstract

This paper studies the evolution of two clusters of firms competing on a common market. Firms exit and enter a cluster based on the perceived chances for profits inside and outside the cluster. Information about profits are diffused by direct communication between firms. Internal and external spillover effects reduce the overall costs of firms in the clusters depending on the number of firms in the own and the competing cluster. A discrete time deterministic dynamical system describing the evolution of cluster sizes is derived. An analysis of the long run attractors of the system and their basins of attraction is used to compare the effects of advantages of a cluster with respect to the size of internal and external spillover effects, respectively. Furthermore, the implications of slow and fast exit and entry behavior of firms for the long run survival and the size of the clusters are studied.

JEL classification: D62; D83; L52

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1. Introduction

In his early contribution Marshall (1920) gives three different reasons why clusters of firms persist. First, firms locate near one another to decrease transportation costs. Second, firms locate near one another so that workers can move from one firm to another in the event of a firm specific downturn. Third and finally, firms locate near

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one another because of intellectual spillovers. In a widely cited paper, Krugman (1991) develops a model based on the interaction of transportation costs and economies of scale that shows how an industrialized core and an agricultural periphery can develop endogenously. In a more recent paper Quah (2000) introduces a model to explain the observed clustering where transportation costs do not matter, but clusters appear as a result from a trade-off between productivity spillovers and a transient stickiness in the factor input location. In this paper, we also focus on the third of Marshall's theories and analyze the intimate relationship between knowledge spillovers and cluster development.

Globalization has dramatically reduced the cost of transporting not just material goods but also information across distances. Many activities of firms in our information-based and dynamic economy take place in low cost locations. Tacit knowledge, on the other hand, cannot easily be transferred across geographic distance. Hence, knowledge spillovers tend to be spatially restricted and many industries are characterized by clustering of firms related to this industry in relatively small geographical regions. Empirical evidence suggests that location and proximity are important factors in exploiting knowledge spillovers (see Head et al., 1995; Audretsch and Feldman, 1996; Ellison and Glaeser, 1997).[1] Porter (1990, 1998) develops a coherent theory which gives clusters a prominent role. He describes how clusters effect competition, productivity and innovation activities. Moreover, he analyzes how clusters develop, the role of the government and the management of firms in this process and also relates his cluster theory to the factors influencing the decision of corporations to locate in a certain region. As one of the main driving forces behind cluster development, Porter identifies the ability of firms within a cluster to capture externalities and spillovers and emphasizes that clusters can affect the productivity of other clusters as well. In other words, not only spillovers within a cluster, but also between clusters are important for cluster development. With regard to this latter point, empirical evidence exists. Head et al. (1995) show that internal spillovers within each group of Japanese and American car manufacturers in the US are larger than external spillovers between these groups. Ellison and Glaeser (1997) find in their empirical study that within-county spillovers are stronger than nearby-county spillovers. Mansfield (1988) demonstrates that the size of internal and external spillovers differ between different regions and industries.

With respect to (government) policy at the cluster level, this raises the question of the effect of creating comparative advantages of a local cluster in utilizing internal and external spillovers. Empirical studies of effects of various public policies concerning R&D and knowledge diffusion have been carried out for numerous industries (see e.g. Movery and Nelson, 1999, for a recent contribution). Here, we take a different approach by considering a highly stylized dynamical model of two competing clusters to address policy questions like: what are the conditions in terms of the difference between internal and external spillovers that allow an initially marginal cluster to coexist with another

[1] In a recent paper Dumais et al. (1997) find empirical support for each of the three theories, where the location process appears to be dominated by the labor mix of a particular area. They mention, however, that this effect could potentially be occurring because industries with similar labor mixes share ideas as well as workers and part of their labor effect is actually attributable to some form of intellectual spillover instead.

cluster of (e.g. foreign) firms in the market? Under which circumstances may internal and/or external spillover effects lead to a market takeover by one cluster? Furthermore, is an increase in internal or external spillover effects more advantageous from the long run perspective of a local industry group?

To answer these questions we carry out a dynamic evolutionary analysis of the competition between two industry groups (one might think of Japanese and US car manufacturers or software companies in the Bay and Boston area) under the presence of internal and external spillover effects. The size of the two industry groups depends on the attractivity of the market in which both groups compete, where attractivity is expressed by the profits achieved by firms who are producing for the market in relation to the profit of some outside option. The market entry and exit decision of firms are made on the basis of information about the relative profitability of the market which has been collected via direct communication within the group. The evolution of the sizes of the two firm clusters for such dynamics has been analyzed in Bischi et al. (2002) in a framework where internal spillover effects within a cluster decreases overall costs for a member of this group, but there are no spillover effects between clusters. It has been shown there that increasing internal spillover effects make it easier for relatively small clusters to stay in the market in the long run—even if spillover effects increase symmetrically in both groups. If one of the two clusters gains increasing advantages with respect to the size of internal spillovers, the effects of these asymmetry are discontinuous. The size of initial market shares which lead to the long run survival of a cluster change abruptly as the advantages in internal spillover effects cross certain thresholds. Here we will build on these results and concentrate on the question of what the implications of differences in the size of external spillover effects between the two clusters are. We will characterize the evolution of the qualitative long run properties of the model as the knowledge transfer stream between the two clusters becomes asymmetric. Furthermore, we will discuss the implications of high flexibility in an industry which leads to fast exit and entry behavior of firms. Here the long run implications of differences in spillover effects are almost impossible to predict and providing generally valid policy recommendations becomes virtually impossible.

The paper is organized as follows. In Section 2 we introduce the model. Section 3 compares the effects of advantages in external and internal spillovers for slow dynamics. Section 4 discusses the implications of fast switching behavior dynamics. Section 5 briefly summarizes the main findings. All proofs are given in Appendix A.

2. The model

Consider two groups of firms $i = 1, 2$ which have to decide whether to produce for a certain market or not. Alternatively, you might think of two groups of investors who have to decide whether to invest in a certain local industry branch or not. As laid out in the introduction, the assignment into one of the groups may be, e.g., due to the national origin of the firms or the current location. To keep matters as simple as possible we assume that the two groups are of the same size and denote by x_{it} the fraction of firms in population i, $i = 1, 2$, which are in the market at time t. This fraction might as well be interpreted as the fraction of available capital in each country which is invested in firms in the market. Assuming constant returns to scale, it does not make a difference for any of our arguments whether output within one country is produced by several large or many small firms as long as no firm has relevant market power.

For reasons of simplicity it is assumed that every firm in the market produces one unit of a homogeneous good per period. Aggregate output in the market is then given by (x_1+x_2) times the number of firms. The market clearing price is determined by an inverse demand function

$$p = p(x_1 + x_2).$$

Due to internal spillovers within the group, overall costs are lower the more other firms in the population produce the same good. Additionally, there are external spillovers and cost reductions also arise due to production activities in the other population. We include such cost externalities in our model by assuming that the unit costs for a firm depend on the number of firms from both populations which produce the good in the market. We express unit costs of a firm in population i by $c_i(x_1,x_2)$, where $\partial c_i/\partial x_j \leqslant 0,\ i,j=1,2,\ \partial c_i/\partial x_i \leqslant \partial c_i/\partial x_j,\ i,j=1,2,\ i \neq j$. Hereby we assume that internal spillover effects are always stronger than spillover effects between populations.

This gives a per period profit of

$$\pi_i(x_1,x_2) = p(x_1+x_2) - c_i(x_1,x_2).$$

The profit of a firm which stays out of the market (i.e. chooses the outside option) is modeled as a stochastic variable. Outside profit of firm f in population i at time t is $u^i_{f,t}$ with expected value U_i. Outside profits are independent across individual firms in a population and time. We write $u^i_{f,t} = U_i + \varepsilon^i_{f,t}$, where the density of $\varepsilon^i_{f,t}$ is independent of f and t, has full support $\mathbb{R}$ and is unimodal and symmetric with respect to 0. The distribution function of $\varepsilon^i_{f,t}$ is denoted by Θ_i.

In each period $t=0,\ldots,\infty$ every firm decides whether to enter or to exit the market.[2] We take an evolutionary approach here and assume that the firm due to the complexity of the environment and a lack of information cannot act as a maximizer of the future discounted profit stream who has perfect foresight.[3] Rather, it is assumed that firms show imitative behavior based on information about profitability of the different options they can collect in their own population.[4] The collection of information is modeled in the simplest possible way: every period every firm is able to observe the action

[2] As indicated above, this can alternatively be interpreted as the decision of an investor in the population to invest in this market by founding a production firm or to withdraw capital, respectively.

[3] Hopenhayn (1992) provides an analysis of long run equilibrium behavior in a similar exit/entry model—in the absence of externalities though—where firms have perfect foresight and maximize expected discounted profits.

[4] Collecting information from the other population might, on one hand, be more difficult if it is assumed that these firms are situated in different regions, and, on the other hand, less informative since the size of the spillover effects, in general, vary significantly between the populations.

and profit of one other firm in the previous period where every other firm in the population is sampled with an identical probability. If a firm samples another firm which has chosen a different action in the previous period, it switches (i.e. exits or enters) whenever the profit of the other firm has been larger than its own profit. When making this decision the firm ignores information it might have collected in previous periods because the relative profitability of production in the market depends crucially on the number of firms which are currently in the market. Past observations which might correspond to very different market conditions, therefore, have little significance for an estimation of the current relative profitability of the two options. The sampling procedure is assumed to be stochastically independent from the outside profit $u^i_{f,t}$ and this implies that the probability that an arbitrary firm in population i which is now in the market exits after period t is

$$\begin{aligned} p^i_{\mathrm{out}}(x_1,x_2) &= (1-x_i)\mathbb{P}(\pi_i(x_1,x_2) < U_i + \varepsilon^i_{f,t}) \\ &= (1-x_i)(1-\Theta_i(\pi_i(x_1,x_2)-U_i)). \end{aligned}$$

On the other hand, a firm currently outside the market enters the market with probability

$$\begin{aligned} p^i_{\mathrm{in}}(x_1,x_2) &= x_i\mathbb{P}(\pi_i(x_1,x_2) > U_i + \varepsilon^i_{f,t}) \\ &= x_i\Theta_i(\pi_i(x_1,x_2)-U_i). \end{aligned}$$

The expected fraction of population i firms ($i = 1,2$) in the market is therefore

$$\begin{aligned} x_{i,t+1} &= x_{i,t} + (1-x_{i,t})p^i_{\mathrm{in}}(x_{1,t},x_{2,t}) - x_{i,t}p^i_{\mathrm{out}}(x_{1,t}x_{2,t}) \\ &= x_{i,t} + x_{i,t}(1-x_{i,t})(\Theta_i(\pi_i(x_{1,t},x_{2,t})-U_i) \\ &\quad - (1-\Theta(\pi_i(x_{1,t},x_{2,t})-U_i)) \\ &= x_{i,t} + x_{i,t}(1-x_{i,t})G_i(\pi_i(x_{1,t},x_{2,t})-U_i), \end{aligned} \tag{1}$$

where $G_i(x) := 2\Theta_i(x) - 1$. The evolution of the fractions x_1 and x_2 are hence described by a nonlinear deterministic system in discrete time. Word of mouth dynamics similar to this one have been analyzed by Ellison and Fudenberg (1993, 1995) and Dawid (1999). It is obvious that the shape of the function G_i depends on the distribution function Θ_i. However, from the fact that Θ_i is a distribution function and the properties of the corresponding density (unimodality and symmetry), it is easy to derive the following statements for $i = 1,2$:

$$G_i(0) = 0, \quad \lim_{x\to\infty} G_i(x) = 1, \quad \lim_{x\to-\infty} G_i(x) = -1.$$

Furthermore, $G_i(x)$ is symmetric with respect to 0, convex on $(-\infty,0]$ and concave on $[0,\infty)$. The slope of G_i at 0 is twice the altitude of the hump of the unimodal density. It will turn out that the qualitative properties of the long run behavior of the dynamics in many cases crucially depend on the ‘speed’ of the flow towards the action with the higher expected profit. Hence, we will use $G_i'(0)$ as a measure of this speed and denote it by λ_i.[5]

Studying the nonlinear two-dimensional dynamical system (1) allows us to derive qualitative features of the evolution of the fraction of firms of the two populations which are in the considered market. In particular, we are interested in the question of how initial market shares of firms of the two populations, $x_{1,0}$ and $x_{2,0}$, and differences in (internal and external) spillovers influence the convergence properties of the evolutionary process to some long run equilibrium (the agglomeration pattern). To answer this question, we will provide an extensive analysis of the equilibria and their basins of attraction. However, before we proceed we need to be more specific about the functions involved.

We will assume that the demand curve is linear:

$$p(x_1 + x_2) = P_0 - B(x_1 + x_2).$$

Furthermore, we use the following rational expression for the unit costs of a firm in population i:

$$c_i(x_1, x_2) = \frac{C_i}{1 + \beta_i x_i + \gamma_i x_j}, \quad i, j \in \{1, 2\}, \ i \neq j.$$

The parameter β_i incorporates the effect of internal spillovers, whereas γ_i characterizes in how far spillovers occur externally between the two populations. As explained in the introduction, the qualitative properties of the cost function are inspired by existing theoretical and empirical works. To take account of the fact that internal spillovers are stronger than those between populations (see Ellison and Glaeser, 1997; Head et al., 1995), we assume that $\beta_i \geqslant \gamma_i$. The profit of a firm in population i which is in the market is then given by

$$\pi_i(x_1, x_2) = P_0 - B(x_1 + x_2) - \frac{C_i}{1 + \beta_i x_i + \gamma_i x_j}, \quad i, j = 1, 2, \ i \neq j. \tag{2}$$

We will always assume that if all firms from both populations are in the market, the payoff for firms in the market is smaller than the expected outside profit. This assumption rules out the rather unrealistic and uninteresting case where spillover effects are so large and the market is so much more attractive than the outside option that all firms from both populations want to enter or stay under all circumstances. Hence, we assume that $\gamma_i \leqslant \beta_i < \bar{\beta}_i$ where

$$P_0 - 2B - \frac{C_i}{1 + 2\bar{\beta}_i} = U_i, \quad i = 1, 2. \tag{3}$$

On the other hand, the profit of the firm first entering the market should be larger than the expected outside profit. This condition is represented by

$$P_0 - C_i > U_i, \quad i = 1, 2. \tag{4}$$

[5] For many classes of distribution functions, like the normal distribution, a large slope of G_i at zero corresponds to a small variance of the outside profit; for example, for the normal distribution λ_i is inversely proportional to σ.

Additionally, we make the more technical assumption that, if there were no spillovers, the expected profit of the outside option would be higher than the profit in the market if half of the firms are in the market

$$P_0 - B - C_i < U_i, \quad i = 1, 2. \tag{5}$$

By making this assumption we avoid the discussion of several cases. However, it would be straightforward to extend the analysis to cases where this assumption does not hold.

Finally, we assume that if a firm enters the market, the marginal negative price effect always outweighs the marginal positive effects from external spillovers for firms in the competing cluster. This is ensured by

$$B > \gamma C_i, \quad i = 1, 2. \tag{6}$$

Using the expressions given above, we obtain the following evolutionary model which describes the dynamics of the fraction of firms from both populations in the market:

$$\begin{aligned} x_{1,t+1} &= x_{1,t} + x_{1,t}(1 - x_{1,t})G_1\left(A_1 - B(x_{1,t} + x_{2,t}) - \frac{C_1}{1 + \beta_1 x_1 + \gamma_1 x_2}\right), \\ x_{2,t+1} &= x_{2,t} + x_{2,t}(1 - x_{2,t})G_2\left(A_2 - B(x_{1,t} + x_{2,t}) - \frac{C_2}{1 + \beta_2 x_2 + \gamma_2 x_1}\right), \end{aligned} \tag{7}$$

where $A_i := P_0 - U_i$. We define $T : [0,1]^2 \mapsto [0,1]^2$ as the right-hand side of (7) and using this notation the system reads $x_{t+1} = T(x_t)$.

In general terms, we have derived a two-population evolutionary model with nonlinear payoff functions and inter- and intra-population interaction. Obviously, the state space $\mathscr{S} := [0,1] \times [0,1]$ is invariant under the dynamics (7). As has to be expected for a nonlinear system like this, we will show that for different parameter constellations there exist several coexisting fixed points x^*. We call the set $\mathscr{B}_T(x^*) = \{x \mid \lim_{t\to\infty} T^t(x) = x^*\}$ the basin of attraction of the fixed point x^* for the mapping T. For an isolated fixed point x^* the basin of attraction is of positive measure if and only if the point is locally asymptotically stable. Standard arguments used in the evolutionary games literature (see e.g. Weibull, 1995) establish that every equilibrium of the market game is a fixed point of the map T. Additionally, the map T may have fixed points in $[0,1]^2$ which are no equilibria. However, none of these can be locally asymptotically stable. This means that every fixed point x^* of T where $\mathscr{B}_T(x^*)$ has positive measure has to be an equilibrium of the underlying economic system. In the following two sections we will characterize how the set of equilibria changes as the ratios of the spillover parameters of the two populations change. Furthermore, we will also describe the transition of the basins of attraction of the coexisting equilibria. Hence, we will provide a rather complete description of the relationship of initial and long run cluster sizes which allows us to address the research questions given in the introduction.

To keep our exposition as clear and simple as possible and in order to focus on the role of spillovers, we assume that the constant unit costs of a single firm from either population are identical: $C_1 = C_2 = C$. In other words, the profit a single firm can achieve when entering an empty market is independent of the population the firm belongs to.

Furthermore, we assume that the distribution of the outside profit is identical in both populations, i.e. $\Theta_1 = \Theta_2$, which in particular implies $G_1 = G_2 := G$ and $U_1 = U_2$. Therefore, we have $A_1 = A_2 = A$. The populations might differ, however, with regard to their infrastructure facilitating spillovers and cost externalities between their members (i.e. with respect to β_i and γ_i).

3. Asymmetric spillovers and their long run effects

In this section we will analyze the long run properties of the system under the assumption that the market exit and entry behavior of firms is so slow that qualitatively the system behaves like the corresponding continuous time system. In particular, let us assume that the map T is invertible (it is easy to see that T is invertible on $[0,1]^2$ for sufficiently small $\lambda_1 = \lambda_2 := \lambda$) and no local overshooting occurs. This corresponds to a situation where the variance of the outside profit is large. In the following section we will then discuss how far these findings change if we allow for larger step sizes and overshooting effects.

The implications of asymmetries in internal spillover effects in the absence of external spillovers have been studied in detail in Bischi et al. (2002). Here we will concentrate on the effects of asymmetries in the ability to transfer knowledge from outside into a firm cluster. In particular, we will also compare advantages in this respect to asymmetries in internal spillover effects. The assumption of asymmetric external spillovers is motivated by empirical evidence. Mansfield (1988) found in his study that countries differ with regard to their ability to adopt and use foreign technology. Whereas US and Japanese firms were comparable in exploiting internally developed technologies, the empirical evidence revealed a big difference in the use of externally based technologies. Japanese firms pursue foreign technology more aggressively and efficiently than their rivals. Accordingly, there is evidence for an asymmetry in spillovers. Several sources of external spillovers can be named, including transfers of knowledge between two countries (due to, e.g., an exchange of engineers, managers and workers) and direct foreign investment. See Chuang and Lin (1999), who identify foreign direct investment as the major channel of technology transfer from multinational enterprises to domestic firms.

To analyze the system we define the curves F_i, $i = 1,2$, as the set of all points (x_1, x_2) where the profit in the market equals the expected outside profit for a firm of population i, i.e.

$$F_i = \{(x_1, x_2) \in [0,1]^2 | \pi_i(x_1, x_2) = U_i\}. \tag{8}$$

Interior equilibria exist at all intersections of the curves F_1 and F_2. Fixed points at the boundary occur either at the intersection of F_1 with $x_2 = 0$ or 1, or at the intersection of F_2 with $x_1 = 0$ or 1. Note, however, that fixed points on the boundary might not correspond to Nash equilibria of the model. The only fixed points of the system which do not lie on one of these curves are the vertices. We denote the vertices by $0 = (0,0)$, $V_{\mathrm{I}} = (1,0)$, $V_{\mathrm{II}} = (1,1)$ and $V_{\mathrm{III}} = (0,1)$. It is easy to see that under our assumptions 0 and V_{II} are unstable and therefore of limited interest. The remaining

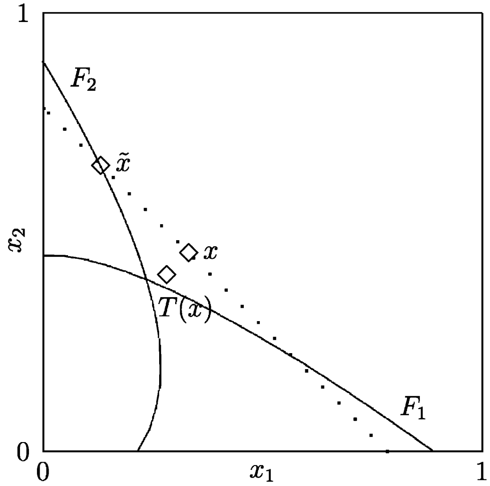

Fig. 1. Illustration of the properties of the equal profit curves F_1 and F_2.

vertices V_{I} and V_{III} correspond to states where the market has been taken over by one cluster which has reached its maximal possible size. Stability of these vertices depends on the parameter constellation. The following lemma summarizes several properties of the equal profit curves F_i which will be useful in the analysis.

Lemma 1. *Under our assumptions the equal profit curves have the following properties*:

(a) *Let* $\tilde{x}$ *be an arbitrary point on* F_1 [*on* F_2]. *Then for every point* x *with* $x_1 + x_2 = \tilde{x}_1 + \tilde{x}_2$, *we have* $(x_1 - \tilde{x}_1)(T_1(x) - x_1) > 0$ $[(x_2 - \tilde{x}_2)(T_2(x) - x_2) > 0]$.
(b) *For* $x_1 \in [0, 1]$ $[x_2 \in [0, 1]]$ *the correspondence* $f_2(x_1) := \{x_2 | (x_1, x_2) \in F_1\}$ $[f_1(x_2) := \{x_1 | (x_1, x_2) \in F_2\}]$ *is either single-valued or empty. Where* f_1 $[f_2]$ *is nonempty we have* $(\partial/\partial\gamma_1) f_2(x_1) > 0, (\partial/\partial\beta_1) f_2(x_1) > 0$ $[(\partial/\partial\gamma_2) f_1(x_2) > 0, (\partial/\partial\beta_2) f_1(x_2) > 0]$.
(c) *The curves* F_1 *and* F_2 *are hyperbolae. The correspondences* $f_1(x_2)$ *and* $f_2(x_1)$ *are concave functions on the range where they are nonempty.*
(d) *The curves* F_1 *and* F_2 *have at most one intersection in* $[0, 1]^2$. *If there is an intersection*, *this fixed point of* T *is either a saddle or a repelling node.*
(e) *For* $\beta_1 = \gamma_1 = 0$ $[\beta_2 = \gamma_2 = 0]$, $F_1[F_2]$ *has exactly one intersection with the line* $\{0\} \times [0, 1]$ $[[0, 1] \times \{0\}]$.

The lemma is illustrated in Fig. 1 where we depict a typical pair of equal profit curves F_1, F_2.

We can study the attractors and basins of attraction of our dynamic model by analyzing the changes in F_1 and F_2 as β_i and γ_i change. We start with a scenario where both internal and external spillover effects are weak and symmetric between the two populations and describe the transition as the firm cluster in population one is able to gain advantages either in the internal or the external spillover parameters. All our statements are formulated for the curve F_1 but symmetric results hold for F_2. It can be easily seen (see Bischi et al., 2002) that for

$$\beta_1 < \hat{\beta} := \frac{C}{A-B} - 1 > 0,$$

the curve F_1 has an intersection with the line segment $[0,1] \times \{0\}$. We call this point $P_{\mathrm{I}} = (p_{\mathrm{I}1}, 0)$. For symmetric internal spillovers $\beta_2 = \beta_1$ we also have an intersection $P_{\mathrm{IV}} = (0, p_{\mathrm{IV}2})$ between F_2 and $\{0\} \times [0,1]$ where $p_{\mathrm{IV}2} = p_{\mathrm{I}1}$. Point (a) of Lemma 1 shows that if we draw a straight line with slope-1 through an arbitrary point of F_1 (F_2), this line cannot have a second intersection with the curve F_1 (F_2). In particular, this implies that F_1 has to intersect $\{0\} \times [0,1]$ between 0 and P_{IV} (F_2 has to intersect $[0,1] \times \{0\}$ between 0 and P_{I}) regardless of γ_1 and γ_2. Hence, as long as we have symmetric internal spillovers with $\beta_1 = \beta_2 < \hat{\beta}$ the model always has three equilibria, namely the interior fixed point S, P_{I} and P_{IV}. P_{I} and P_{IV} are the only locally stable fixed points of T, whereas the interior equilibrium S is a saddle point and its stable manifold separates the basins of attraction of P_{I} and P_{IV}. Note that in the equilibrium P_{I} only firms from population 1 are in the market, whereas in P_{IV} only firms from population 2 are in the market. This implies that for small symmetric internal spillovers, always one of the two clusters disappears in the long run. However, the decline of the exiting cluster does not have to be immediate or monotonic. The trajectories of the system first approach the unstable manifold of the interior saddle point S and then follow this manifold to one of the equilibria P_{I} or P_{IV}. Hence, if both clusters are initially small they will both grow for some time until the market becomes tight, and firms where cost reductions due to spillovers are comparably small, start exiting the market bringing down the 'weaker' cluster. If external spillovers are symmetric as well, clearly the basins of attraction of the two stable fixed points are separated by the main diagonal of the unit square, which means that the cluster which is initially larger is able to eventually drive the competing cluster out of the market. If the cluster in population 1 is able to improve its infrastructure in such a way that $\gamma_1 > \gamma_2$, the interior fixed point S and also its stable manifold moves up in the unit square which means that the set of initial cluster sizes which will lead to market domination of cluster 1 increases. Let us point out that the effect of advantages in external spillover effects are continuous and coexistence of clusters can never occur as long as β is small. We illustrate this scenario in Fig. 2 [6] and move on to cases where spillover effects are more pronounced. We will see that this allows for different and more interesting long run patterns.

As β becomes larger than $\hat{\beta}$ (note that $\hat{\beta} < \bar{\beta}$), the two equilibria P_{I} and P_{IV} wander through the vertices V_{I} and V_{III} and disappear. The two stable equilibria are V_{I} and V_{III} then, which means that the cluster which takes over the market will grow to its maximal possible size. The curve F_1 has no intersection with $[0,1] \times \{0\}$ anymore

[6] In the numerical illustrations we always use the specification $G(x) = (2/\pi)\arctan((\lambda\pi/2)x)$, where $\lambda = \lambda_1 = \lambda_2$. This function satisfies all the assumptions for the switching function G and $\lambda = G'(0)$.

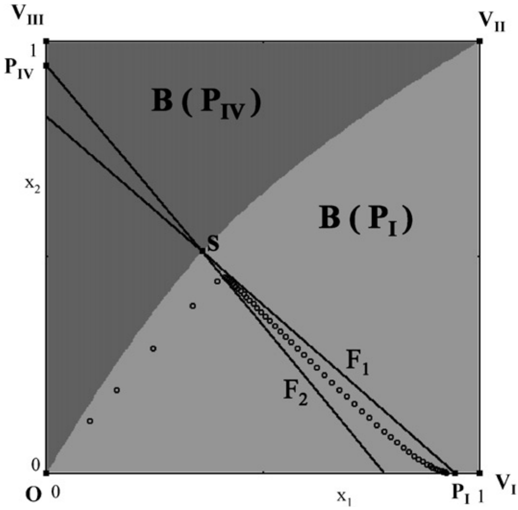

Fig. 2. Scenario with small internal and external spillovers: the two co-existing stable equilibria P_{I} and P_{IV} with their basins of attraction and a trajectory converging to P_{I} ($P_0 = 300$, $B = 100$, $C = 190$, $U = 32$, $\beta = 0.1$, $\gamma_1 = 0.03$, $\gamma_2 = 0$).

but this intersection point has moved to $\{1\} \times [0, 1]$. We will take such a situation with symmetric internal and external spillovers and two coexisting stable equilibria V_{I} and V_{III} as a starting point to compare the effects of unilateral increases of β_i and γ_i. We know from Lemma 1 (b) that the curve F_1 moves upwards whenever either β_1 or γ_1 is increased. Eventually, this might lead to a collision of F_1 with $[0, 1] \times \{1\}$ and to the creation of additional fixed points of the dynamical system. If this collision occurs to the left of the intersection of F_2 with $[0, 1] \times \{1\}$, a new stable equilibrium is created and the long run properties of the process change significantly. In the following proposition we show that the implications of an increase in β_1 and γ_1 are, however, slightly different:

Proposition 2. *Consider a scenario with symmetric internal and external spillover effects* $\beta_1 = \beta_2 = \beta^{\mathrm{I}} > \hat{\beta}, \gamma_1 = \gamma_2 = \gamma^{\mathrm{I}} \leqslant \beta^{\mathrm{I}}$ *where no fixed points other than the vertices and* S *exist.*

(a) *There exists a value* $\gamma_1^*(\beta^{\mathrm{I}}) < \beta^{\mathrm{I}}$ *such that when* γ_1 *is increased and crosses* $\gamma^*(\beta^{\mathrm{I}})$, *either a pair of fixed points* $Q_{\mathrm{III}} = (q_{\mathrm{III}1}, 1), P_{\mathrm{III}} = (p_{\mathrm{III}1}, 1), p_{\mathrm{III}1} > q_{\mathrm{III}1}$ *is created on* $[0, 1] \times \{1\}$ *or a fixed point* P_{III} *enters this line from the vertex* V_{III}. *Either* P_{III} *is a locally asymptotically stable equilibrium and* Q_{III} *is a saddle point* (*if it exists*) *or* P_{III} *is a saddle point and* Q_{III} *a repelling node.*
(b) *If there exists a value* $\beta_1^*(\gamma^{\mathrm{I}}) < \bar{\beta}$ *such that a pair of fixed points* P_{III} *and* Q_{III} *with properties like in* (a) *are created when* β_1 *crosses* $\beta_1^*(\gamma^{\mathrm{I}})$, *then we always have* $\gamma^*(\beta^{\mathrm{I}}) - \gamma^{\mathrm{I}} < \beta^*(\gamma^{\mathrm{I}}) - \beta^{\mathrm{I}}$.

(c) *Whenever a unilateral increase of* β_1 *beyond* $\beta_1^*(\gamma^{\mathrm{I}})$ *creates a* locally asymptotically stable *equilibrium* $P_{\mathrm{III}} \in [0,1]\times\{1\}$, *a unilateral increase of* γ_1 *beyond* $\gamma_1^*(\beta^{\mathrm{I}})$ *creates a locally asymptotically stable equilibrium as well* (*but not vice versa*).

Mathematically speaking, the crossing of F_1 with $[0,1] \times \{1\}$ triggers a tangent bifurcation (see Lorenz, 1993). If the new equilibrium P_{III} is stable it has a basin of attraction of positive size as soon as it appears. This basin is bounded by the stable manifolds of the two saddle points Q_{III} and S. Thus, in economic terms a unilateral increase of either γ_1 or β_1 initially has continuous effects in the sense that the sets of initial cluster sizes which lead to market dominance of cluster 1 first increases continuously. However, as soon as one of the thresholds γ_1^* or β_1^* are crossed we get an abrupt qualitative change of the long run properties. For all initial cluster sizes between the stable manifolds of S and Q_{III}, now the cluster in population 1 is able to survive in the long run although the cluster in population 2 is always larger. This means that if the advantages of population 1 with respect to spillover effect becomes larger than a certain threshold, this population suddenly is able to survive in the market even if the cluster in the other population has a rather large advantage in market share. Interestingly, in his paper, Krugman (1991) mentions that such effects are of empirical relevance. He writes, after describing the effects of transportation costs and increasing returns on the geographical concentration: "This not entirely imaginary history suggests that small changes in the parameters of the economy may have large effects on its qualitative behavior. That is, when some index [...] crosses a critical threshold, population will start to concentrate and regions to diverge; once started, this process will feed on itself." (p. 487).

Comparing the effects of increases in γ_1 and β_1 we can clearly see that increasing γ_1 is more effective. Whenever an increase in β_1 can create a new equilibrium where cluster 1 is able to stay in the market as the smaller cluster, such an equilibrium can also be created by an increase in γ_1, and the increase in γ_1 needed is always smaller than that needed in β_1. To get a better economic interpretation of this fact we observe that

$$\gamma_1 = -\frac{C_1 c_{1x_2}(x_1,x_2;\beta_1,\gamma_1)}{c_1(x_1,x_2;\beta_1,\gamma_1)^2}, \qquad \beta_1 = -\frac{C_1 c_{1x_1}(x_1,x_2;\beta_1,\gamma_1)}{c_1(x_1,x_2;\beta_1,\gamma_1)^2}.$$

Using this, it can be easily derived that $\gamma_1^* - \gamma_1 < \beta_1^* - \beta_1$, implies that

$$\frac{c_{1x_2}(x_1,x_2;\beta_1,\gamma_1)}{c_1(x_1,x_2;\beta_1,\gamma_1)} - \frac{c_{1x_2}(x_1,x_2;\beta_1,\gamma_1^*)}{c_1(x_1,x_2;\beta_1,\gamma_1^*)} < \frac{c_{1x_1}(x_1,x_2;\beta_1,\gamma_1)}{c_1(x_1,x_2;\beta_1,\gamma_1)} - \frac{c_{1x_1}(x_1,x_2;\beta_1^*,\gamma_1)}{c_1(x_1,x_2;\beta_1^*,\gamma_1)}$$

holds for $x_1 = x_2$. Note that $c_{1x_2}(x_1,x_2;\beta_1,\gamma_1)/c_1(x_1,x_2;\beta_1\gamma_1)$ gives the relative marginal cost effect for cluster 1 firms of an increase of cluster 2, whereas $c_{1x_1}(x_1,x_2;\beta_1,\gamma_1)/c_1(x_1,x_2;\beta_1,\gamma_1)$ gives the relative marginal cost effect for cluster 1 firms of an increase of cluster 1. Our result says that in order to create the equilibrium Q_{III}, the required increase in the marginal cost effect of one additional firm in the market is smaller if it is targeted at foreign firms rather than domestic ones. Of course, it is still debatable whether such an increase is equally feasible if external rather internal spillovers

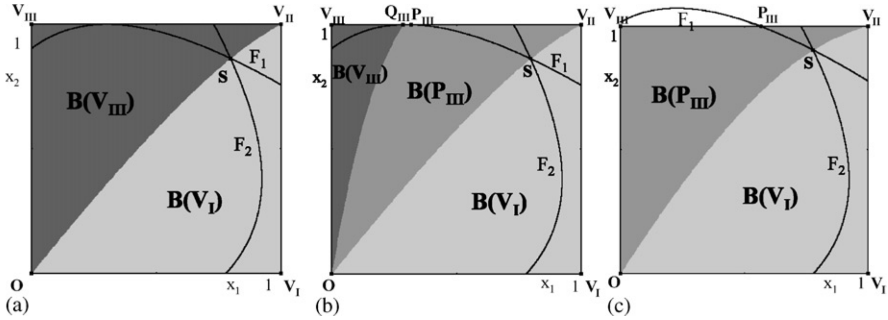

Fig. 3. Effects of advantages in the size of external spillovers with slow dynamics: (a) $\gamma_1 = 0.076$; (b) $\gamma_1 = 0.077$; (c) $\gamma = 1.2$ ($P_0 = 300,\ B = 100,\ C = 190, U = 32,\ \beta = 1,\ \gamma_2 = 0$).

are involved, and clearly our model is far too abstract to address this question. Note also that (b) does not say that an increase of β_1 within the admissible range $[0, \bar{\beta}]$ always leads to the creation of additional equilibria. On the other hand, an increase of γ_1 within the admissible range always leads to the creation of additional fixed points (they might be unstable however).

In cases where no saddle point Q_{III} exists on $(0,1) \times \{1\}$ (because either P_{III} wandered through V_{III} at $\gamma_1 = \gamma^*$ or Q_{III} wandered through V_{III} as γ_1 was increased further) all initial conditions left of the stable manifold are attracted by P_{III}. Hence, if γ_1 is sufficiently increased that such a scenario is reached, the cluster in population 1 survives regardless of the initial ratio of cluster sizes.

In Fig. 3 we illustrate a transition of the type described above. Fig. 3a shows the basins of the two stable equilibria V_{I} and V_{III} for symmetric values of β and slightly asymmetric external spillovers. Fig. 3b shows the basins of attraction, after γ_1 has been increased above γ_1^* and the pair of equilibria P_{III} and Q_{III} has been created. A similar picture could have been produced by increasing β_1 but the necessary increases would have been larger. If the value of γ_1 is further increased, the saddle point Q_{III} moves through the vertex V_{III} and the vertex becomes unstable (due to a transcritical bifurcation, where a change of stability occurs). After this bifurcation there are only two stable equilibria namely P_{III} and V_{I}. In other words, regardless of the initial market shares, firms from population 1 are never completely driven out of the market. This situation is illustrated in Fig. 3c. It is interesting to note that if β_1 were increased instead of γ_1, there would always be a set of initial cluster sizes where the cluster in population 1 vanishes. This is easy to see, since the intersection point of F_1 with $\{0\} \times [0,1]$ does not change for increasing β_1 and therefore always stays below V_{III}. This shows that population 1 has to create sufficiently large *external* spillover effects, in order to guarantee the survival of a firm cluster regardless of the initial relative size.

A further increase of γ_1 finally leads to a collision of the interior fixed point S with the boundary fixed point P_{III} (with a corresponding exchange of stability) and after that the only stable fixed point is V_{I}. Thus, population 1 takes over the entire market regardless of the initial market conditions. Note that, again, we have an instantaneous change of the long run behavior of the system. If γ_1 is only slightly smaller than the bifurcation value, there is still a set of initial market constellations with positive and often significant measure which lead to long run market participation of all population 2 firms. As soon as γ_1 crosses this value and is slightly larger, all firms from population 2 leave the market and choose the outside option for all those initial states. For different parameter settings these two transitions might occur in reversed order, however, with the same final result.

Symmetry arguments show that a similar transition along the border line $x_1 = 1$ occurs if external spillovers from population 1 to population 2 are increased. Hence, there are four coexisting locally stable fixed points if external spillovers in both populations are identical and sufficiently large. In contrast to the case where only internal spillovers exist, a further increase of $\gamma_1 = \gamma_2$ leads to the disappearance of the two stable equilibria on the vertices. Therefore, only two stable equilibria are left, one in the interior of the upper edge of the unit square and one in the interior of the right edge of the unit square. Accordingly, our analysis yields a very intuitive result: large transfers of knowledge between the two populations (due to large external spillovers) result in the long run participation of firms from both populations in the market.

This concludes our discussion of the effects that comparative advantages in internal and external spillovers have on the long run evolution of the cluster in population 1 under the assumption that exit and entry behaviors of firms in both populations is slow and the system evolves in small step sizes. The flexibility of firms and the speed of exit and entry, however, differs significantly between industries. In particular, for the study of branches of the 'new economy' it should be analyzed whether the qualitative insights obtained in this analysis can be upheld if fast dynamics are considered.

4. Fast dynamics and basin bifurcations

In the description of the effects of an increase of γ_1, so far we have considered slow dynamics. In particular, in our discussion we have implicitly assumed that the stable manifolds of the saddle points separating the basins of attraction are never overshot by the dynamics which means that they are forward and backward invariant. Hence, we were dealing with connected basins of attraction of all equilibria. In Fig. 4 we show the basins of attraction of the stable equilibria for increasing γ_1, however this time for fast switching behavior of the agents (i.e. a large value of λ; recall that this corresponds to a small variance of the outside option). We start with the symmetric case where the only stable equilibria are V_{I} and V_{III} and unilaterally increase γ_1. Fig. 4a shows the situation where γ_1 is slightly smaller than $\gamma^*(\beta^{\mathrm{I}})$. Again we can see that this increase had a continuous effect by slightly enlarging the basin of V_{I}. After γ_1 crosses γ^* the situation becomes, however, much more complex (see Fig. 4b). The pair of new fixed points $P_{\mathrm{III}}, Q_{\mathrm{III}}$ emerges—as discussed in the previous section—where P_{III} is locally asymptotically stable. But the basin of attraction of P_{III} has quite a complex structure since it is intermingled with the basin of V_{III}. Quite obviously the prediction of long run outcomes based on the initial cluster sizes is very difficult and sensitive to small

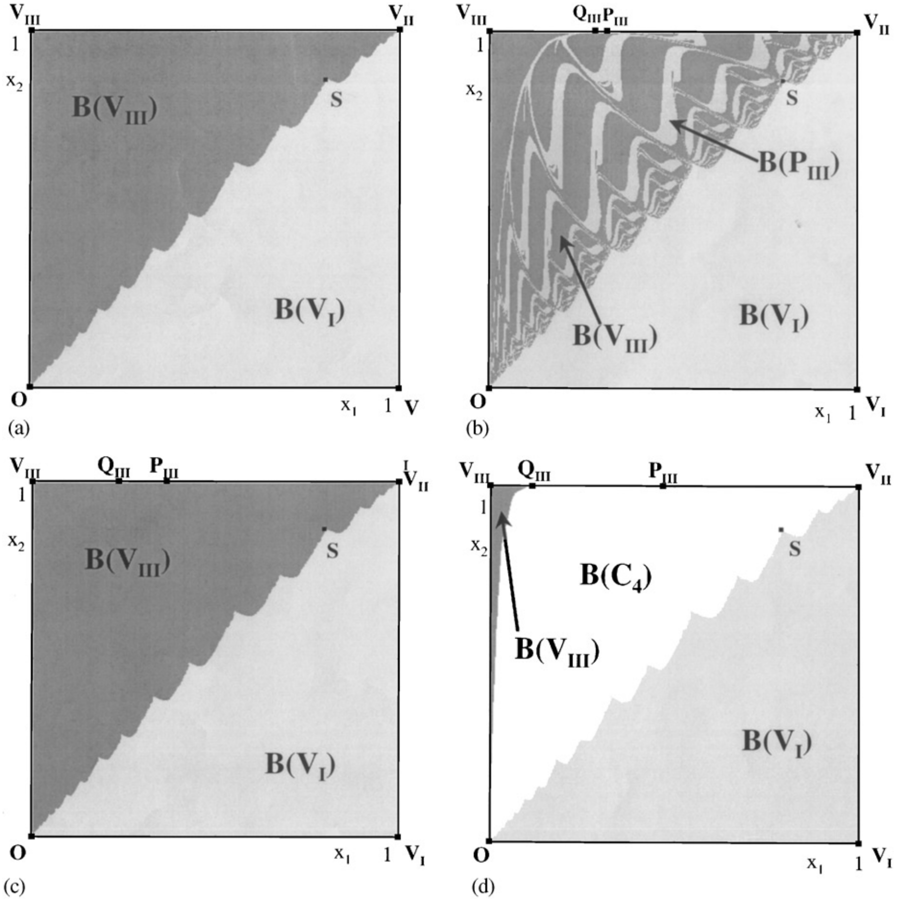

Fig. 4. Transition of the basins of attraction of the locally stable equilibria for fast dynamics, symmetric internal spillovers and increasing positive external spillovers from population 2 to population 1 firms: (a) $\gamma_1 = 0.076$; (b) $\gamma_1 = 0.077$; (c) $\gamma_1 = 0.08$; (d) $\gamma_1 = 0.1$ ($P_0 = 300$, $B = 100$, $C = 190$, $U = 32$, $\beta = 1$, $\gamma_2 = 0$, $\lambda = 7.6/\pi$). The white region in (d) represents the basin of a 4-cycle along the line $x_2 = 1$.

changes in the initial conditions. For a given number of firms in population 1 in the market, an increase in the initial number of firms from population 2 in the market does not necessarily imply a higher long run market share for this population. On the contrary, a higher initial fraction of firms in the market may lead to a long run market share of zero whereas a lower initial fraction leads to the convergence to P_{III} and the long run survival of a firm cluster from population 2 in the market.

If we compare Fig. 4b to Fig. 3b (all parameters but λ are identical in these two figures) we realize that the faster dynamics in Fig. 4b reduces the advantages population 1 can gain from the higher external spillover effects directed towards firms in this cluster. The set of initial conditions where the population survives in the market is substantially smaller in the fast dynamics scenario. Note, however, that the constellation of fixed points and their local stability properties are identical in both figures. Accordingly, and this is important to realize, local analysis cannot be used to explain this change in the long run properties of the process.

In order to understand the occurrence of such a global bifurcation from a mathematical point of view we can employ the theory of critical curves (see e.g. Mira et al., 1996). We believe that this theory is a very useful tool to analyze the global properties of evolutionary and more general dynamic economic models which otherwise, in general, have to be examined by simulation methods. Therefore, we provide a (very) brief sketch of the argument here in order to outline their use. For general definitions and more extensive explanations we refer to Mira et al. (1996) or Abraham et al. (1997). For a more extensive analysis along these lines in a version of this model without external spillover effects the reader should consult Bischi et al. (2002). Critical curves separate areas where the number of (rank-1) preimages of points coincide. Whenever points have different numbers of (rank-1) preimages, there has to be at least one critical curve between these points. If we denote the set of all points where the determinant of the Jacobian of the map T vanishes by LC_{-1}, then the critical curve LC can be determined by applying the map T to all points of this set, i.e. $LC = T(LC_{-1})$. Whereas in this model there are no critical curves in $[0,1]^2$ for small λ (which means that the map T is invertible on the unit square), for increasing λ a closed critical curve surrounding the interior fixed point S appears and expands. The region outside LC is the region Z_1 of points with only one rank-1 preimage, and inside LC there are points with three rank-1 preimages, which we call Z_3. Let us now consider the transition from Fig. 3b to Fig. 4b. If the speed of the dynamics increases from the level in Fig. 3b, eventually a closed critical curve around S appears. Initially, the region Z_3 inside this curve is entirely included in the basins of P_{III} and V_{I}, but as λ is further increased, the critical curve LC and the region Z_3 expands. Due to this expansion, LC eventually contacts the stable set of the fixed point Q_{III}, which constitutes the boundary between the basins of P_{III} and V_{III} (in fact, numerical evidence reveals that the first contact of LC and the boundary which separates the basins occurs along the boundary $x_2 = 1$). After this contact occurred, a small portion of Z_3 enters the basin of V_{III}. This means that suddenly a small portion of the basin of V_{III} has a larger number of preimages, namely three instead of one. The two new rank-1 preimages of this portion merge along LC_{-1}. Since they are inside Z_3 these preimages again have three (rank-1) preimages (which are rank-2 preimages of the small region which have been created when LC crossed the basin boundary). This leads to an arborescent sequence of preimages. All these preimages belong to the basin of attraction of V_{III}, since they are mapped into the immediate basin of V_{III} after a finite number of iterations, and hence we get the islands of the basin of V_{III} within the basin of P_{III} which are observable in Fig. 4b.

We do not regard these findings as mathematical artifacts of our dynamic model, but there is a clear economic mechanism at work, namely the interaction of price fluctuations on the market (which corresponds to fluctuations in the size of cluster 1) and large externalities in population 1. In the absence of externalities the exit and entry dynamics around P_{III}, which is driven by price fluctuations, would result in dampening

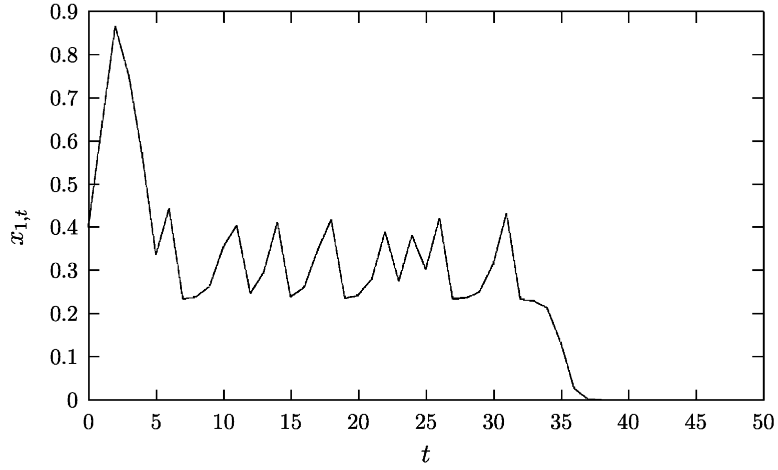

Fig. 5. Evolution of the size of the cluster in population 1 for initial conditions $x_{1,0} = 0.4$, $x_{2,0} = 0.6$. ($P_0 = 300$, $B = 100$, $C = 190$, $U = 32$, $\beta = 1$, $\gamma_1 = 0.8$, $\gamma_2 = 0$, $\lambda = 7.6/\pi$).

oscillations and eventual convergence to P_{III}. However, in our scenario a massive exit of firms of cluster 1 in a period might lead to such a strong increase in the production costs in the cluster (due to the reduced spillover effects) that the increase of the market price is not sufficient to make it profitable to produce for the market. In such a case, cluster 1 is of no viable size anymore and quickly disappears. Basin bifurcations similar to this which lead to intermingled basins of attraction have been observed before in dynamic economic models (e.g. Bischi et al., 2000b; Bischi and Kopel, 2001).

If γ_1 is further increased and the dynamics is sufficiently fast, the fixed point P_{III} becomes unstable along the invariant line $x_2 = 1$ and bifurcates into cyclical attractors of increasing period situated between Q_{III} and V_{II}. These attractors expand until the cyclical or chaotic attractor collides with Q_{III}. After such a collision all trajectories oscillating around P_{III} are eventually mapped into the area left of Q_{III} and end up in V_{III}. Consequently, the only attractors of the adaptation dynamics generated by the map T on the unit square are again V_{I} and V_{III} (Fig. 4c). Interestingly, we get a counterintuitive result stating that the further increase of γ_1 has weakened the position of population 1. The mixed equilibrium has disappeared, and the situation is somehow reminiscent of the scenario before any basin bifurcation has occurred; compare Figs. 4a and c.

Note, however, that although the basins in (a) and (c) look similar, the transient behavior of a trajectory $(x_1, x_2)_t$ for these two parameter settings in general differs significantly. In particular, in case (c) trajectories close to the line $x_2 = 1$ in general oscillate for some time before converging to V_{III}. In Fig. 5 we show the sizes of the cluster in population 1 along such a trajectory. The cluster initially grows, but soon shrinks and is able to survive for a substantial period of time showing persistent oscillations of size before it suddenly disappears.

On the other hand, for low γ_1 the firms in population 1 start leaving the cluster when the market gets tight and after that period, the cluster in population 1 shrinks continuously until it vanishes. So, if we just look at the transient behavior, increasing external spillovers from population 2 to population 1 have the effect of keeping firms from population 1 in the market for a longer period of time, although in both cases they eventually leave the market. This transient effect of a viable population 1 cluster only becomes a long term effect if γ_1 is further increased and scenario (d) is obtained. The fixed point Q_{III} moves towards V_{III} as γ_1 is increased and eventually crosses the critical curve LC again and wanders from the region Z_3 back to the region Z_1. After this has happened, the stable manifold of the saddle point is again forwards and backwards invariant and no trajectory with initial conditions to the right of this boundary can converge to V_{III}. All the trajectories between the stable manifold of Q_{III} and the stable manifold of the interior equilibrium S converge to some attractor on $[0,1]\times\{1\}$ (see Fig. 4d). This attractor might be either cyclical or chaotic, but now the advantages in the spillover size are so large that these oscillations never lead to a shrinking of the cluster below the size it needs for survival.

This example of a transition with fast dynamics shows that not all findings in the previous section obtained using slow dynamics do necessarily hold if switching behavior is fast. Increasing the size of external spillover effects facilitates persistent oscillations of the cluster size if the exit and entry behavior is very flexible. These oscillations bear the danger that the cluster becomes so small that entering is unattractive in spite of the large influx of knowledge from outside (remember that we always assume that spillovers within the cluster are larger than between the clusters). We have seen that due to this effect the implications of an increase in γ_1 can be negative in some cases.

The basin bifurcations in this model always imply that the basin of a vertex invades that of an equilibrium on the boundary. Accordingly, the subsequent change of the structure of the basins always increases the size of the basin of the vertices and strengthens the position of the population with the larger initial market share compared to that with the higher ability to utilize spillover effects. This yields the interesting result that in flexible industries with low exit and entry barriers the effect of initial advantages in cluster size for the long run survival and size of a cluster is larger than in industries with slower exit and entry behavior. In these inert industries differences in the ability of the clusters to create external and internal spillover effects are of higher importance.

These observations might raise the question whether our results from the last section about the comparison of advantages in β_1 and γ_1 change qualitatively if fast dynamics are considered. Of course, the fact that the additional equilibrium P_{III} appears 'earlier' if γ_1 rather than β_1 is increased is independent of the speed of the dynamics. Also, it is easy to see that the equilibrium Q_{III} is further left on $[0,1]\times\{1\}$ for parameters $(\beta^{\mathrm{I}},\gamma_1)$ compared to $(\beta_1,\gamma^{\mathrm{I}})$ where $\beta_1-\beta^{\mathrm{I}}=\gamma_1-\gamma^{\mathrm{I}}$. This suggests that not only advantages in external spillover effects needed to create the market sharing equilibrium P_{III} are smaller but that the expansion of the critical curve needed to create a basin bifurcation—which hurts the population with the spillover advantages—is larger than that for identical advantages in β_1. Strictly speaking, we have to take into account the differences in the critical curves as well, so this is no rigorous formal argument. However, the intuition behind this has been backed up in all the scenarios we have numerically examined. In all cases, advantages in γ_1 yielded a vastly larger set of initial cluster sizes leading to long run survival of the cluster from population 1 than identical advantages in β_1.

5. Conclusions

This study shows that gaining advantages in the size of inter-cluster spillovers are more efficient in gaining long run market presence or market dominance than gaining advantages of comparable size for intra-cluster spillovers. Furthermore, it has been shown that in cases where market exit and entry behavior of firms is fast there is a region of initial cluster sizes where a prediction of the long run size of the clusters is virtually impossible (although we do not take any stochastic outside influences into account). In such situations a policy increasing advantages in external spillovers might have a detrimental effect on the long run size of a cluster. The reason of such unpredictability is not complex long run behavior of the system but the interaction of overshooting at the split market equilibrium with externalities. In very liquid markets, like stock markets, the importance of such overshooting phenomena is well known and with the lowering of entry barriers in new technology industries, like the software industry, the implications of fast switching we have identified here should be of high relevance for the understanding of the effects of industrial policies.

Acknowledgements

The helpful comments of an anonymous referee are gratefully acknowledged.

Appendix A.

A.1. Proof of Lemma 1

(a) Note that along any straight line in $[0,1]^2$ with slope-1 the overall number of firms in the market and, therefore, also the market price stays constant. Since we assume $\beta_i \geqslant \gamma_i$, the profit difference $\pi_i(x_1,x_2) - U_i$ increases along any such line. In particular, this means that if we draw a straight line L with slope-1 through an arbitrary point (x_1,x_2) of F_1, we have $\pi_1(\tilde{x}_1,\tilde{x}_2) > U_1$ for every point $(\tilde{x}_1,\tilde{x}_2)\in L$, such that $\tilde{x}_1 > x_1$ and $\pi_1(\tilde{x}_1,\tilde{x}_2) < U_1$ for every point $(\tilde{x}_1,\tilde{x}_2)\in L$, such that $\tilde{x}_1 < x_1$. The same argument shows that if L is a straight line with slope-1 through a point (x_1,x_2) on F_2 $\pi_2(\tilde{x}_1,\tilde{x}_2) < U_2$ for every point $(\tilde{x}_1,\tilde{x}_2)\in L$, such that $\tilde{x}_1 > x_1$, and $\pi_2(\tilde{x}_1,\tilde{x}_2) > U_2$ for every point $(\tilde{x}_1,\tilde{x}_2)\in L$, such that $\tilde{x}_1 < x_1$.

(b) The curve of equal profit F_1 is given by

$$B\beta_1 x_1^2 + B(\beta_1+\gamma_1)x_1x_2 + B\gamma_1 x_2^2 + (B - A_1\beta_1)x_1 + (B - A_1\gamma_1)x_2 + C_1 - A_1 = 0 \tag{A.1}$$

and the curve F_2 satisfies

$$B\beta_2 x_2^2 + B(\beta_2+\gamma_2)x_1x_2 + B\gamma_2 x_1^2 + (B - A_2\beta_2)x_2 + (B - A_2\gamma_2)x_1 + C_2 - A_2 = 0. \tag{A.2}$$

Let us define $g(x_2;x_1)$ as the left-hand side of (A.1) for a given $x_1 \in [0,1]$. Clearly, the set $f_2(x_1)$ is given by the set of roots of this quadratic equation. The equation can have two positive real roots only if the following two conditions hold:

$$g(0;x_1) = B\beta_1 x_1^2 + (B - A\beta_1)x_1 + C - A > 0,$$

$$g'(0;x_1) = B(\beta_1+\gamma_1)x_1 + B - A\gamma_1 < 0.$$

From the first of these inequalities we get using $A > C$ (see (4))

$$A < Bx_1 + \frac{B}{\beta}.$$

From the second we get using $\gamma < \beta$

$$A > B\left(1+\frac{\gamma}{\beta}\right) + \frac{B}{\beta}.$$

Clearly, these two inequities cannot be fulfilled simultaneously and accordingly g can have either one or no positive real root. Assuming that $f_2(x_1)$ is single-valued at x_1, total differentiation of

$$A - B(f_2(x_1)+x_1) - \frac{C}{1+\beta_1 x_1 + \gamma_1 f_2(x_1)} = 0$$

yields

$$\frac{\partial}{\partial\gamma_1} f_2(x_1) = \frac{C}{B(1+\beta_1 x_1+\gamma_1 f_2(x_1))^2 - \gamma_1 C} x_2 > 0,$$

$$\frac{\partial}{\partial\beta_1} f_2(x_1) = \frac{C}{B(1+\beta_1 x_1+\gamma_1 f_2(x_1))^2 - \gamma_1 C} x_1 > 0$$

due to (6).

(c) For $\beta_1 = \gamma_1$ ($\beta_2 = \gamma_2$), $F1$ (F_2) is given by a pair of straight lines with slope-1. From (A.1) and (A.2) we can see that for $\beta_i \neq \gamma_i$, these curves are given by hyperbolas with centers $K_1 = (-(B + A_1\gamma_1)/B(\beta_1-\gamma_1)), (B + A_1\beta_1)/B(\beta_1-\gamma_1)$ for F_1 and $K_2 = ((B + A_2\beta_2)/B(\beta_2-\gamma_2)), -(B + A_2\gamma_2)/B(\beta_2-\gamma_2)$ for F_2. The slopes of the asymptotes are -1 and $-\beta_1/\gamma_1 < -1$ for F_1 and -1 and $-\gamma_2/\beta_2 > -1$ for F_2. Moreover, for $\beta_1 > \gamma_1$ the center of F_1 is left of $[0,1]^2$, which implies that the curve f_1 is a concave function of x_2. Analogously, due to symmetry, the curve f_2 is a concave function of x_1.

(d) If we draw a line with slope-1 through an intersection point (x_1^*, x_2^*) of F_1 and F_2, all points on F_1 with $x_1 > x_1^*$ have to lie above this line, all points on F_1 with $x_1 < x_1^*$

have to lie below this line. On the other hand, every point on F_2 with $x_2 > x_2^*$ has to lie below the line and any point on F_2 with $x_2 < x_2^*$ has to lie above this line. Therefore, there cannot be a second point of intersection of F_1 and F_2. Since we know that the dynamics along the straight line with slope-1 points away from the equilibrium, obviously the interior equilibrium always has to have at least one unstable manifold. Furthermore, it is easy to realize that a straight line between $(0,0)$ and the equilibrium never intersects either F_1 or F_2. Given our assumptions about the direction of the dynamics at $(0,0)$, it follows that the dynamics points towards the interior fixed point along this line. Accordingly, the interior fixed point either has to be a saddle point or a repelling node with one positive and one negative eigenvalue, where both have absolute values larger than one.

(e) Using the notation of the proof of (b) we get for $\beta_1 = \gamma_1 = 0$: $g(0;0) = C - A < 0$ (due to (4)) and $g(1;0) = B + C - A > 0$ (due to (5)). Together with (c) this implies point (e) of the Lemma. □

Proof of Proposition 2. (a) For $\gamma_1 = \beta^{\mathrm{I}}$ the curve F_1 is a straight line with slope-1. Since we know that it has an intersection with $\{1\} \times [0,1]$ for $\beta^{\mathrm{I}} > \hat{\beta}$, this line has to have an intersection with $[0,1] \times \{1\}$ as well. Using the fact that F_1 is a hyperbola which is upward bending in $[0,1]^2$ and continuously moves upwards as γ_1 is increased (Lemma 1) establishes that there has to be a value $\gamma^*(\beta^{\mathrm{I}}) < \beta^{\mathrm{I}}$ such that either a pair of intersection points $Q_{\mathrm{III}}, P_{\mathrm{III}}$ appear in the interior $\{1\} \times (0,1)$ or F_1 goes exactly through V_{III} yielding a single intersection point P_{III}. Since F_1 moves upwards, it is clear that Q_{III} moves left and P_{III} moves right as γ_1 is further increased. Further, it is straightforward to see that Q_{III} is repelling along the invariant manifold $x_2 = 1$ whereas P_{III} is stable along this line. The stability in the transversal direction depends on whether the point is left or right of the intersection of F_2 with $[0,1] \times \{1\}$.

(b) Assume that there exists a $\beta^*(\gamma^{\mathrm{I}})$ such that F_1 touches $[0,1] \times \{1\}$ for $\beta_1 = \beta^*$. Then at the tangent point $(\tilde{x}_1, 1)$ we have

$$\begin{aligned} 0 &= A - B(1+\tilde{x}_1) - \frac{C}{1 + \beta^{\mathrm{I}}\tilde{x}_1 + \gamma^{\mathrm{I}} + (\beta^{\mathrm{I}} - \beta^*)\tilde{x}_1} \\ &< A - B(1+\tilde{x}_1) - \frac{C}{1 + \beta^{\mathrm{I}}\tilde{x}_1 + \gamma^{\mathrm{I}} + (\beta^{\mathrm{I}} - \beta^*)}. \end{aligned}$$

This implies that for $\beta_1 = \beta^{\mathrm{I}}$, $\gamma_1 = \gamma^{\mathrm{I}} + (\beta^* - \beta^{\mathrm{I}})$, the curve F_1 is above $[0,1] \times \{1\}$ at $(\tilde{x}, 1)$. Since we know that the intersection of F_1 with $x_1 = 1$ is below V_{II} there has to be an intersection of F_1 with $[0,1] \times \{1\}$ at $(\tilde{x}, 1)$ between $(\tilde{x}_1, 1)$ and V_{II} for these parameter values. Since F_1 is continuously moving upwards for increasing γ_1, this shows that $\gamma^*(\beta^{\mathrm{I}}) < \gamma^{\mathrm{I}} + \beta^* - \beta^{\mathrm{I}}$.

(c) Assume that there exists a β^* such that a new pair of fixed points emerges at $[0,1] \times \{1\}$. The equilibrium P_{III} is stable if the pair of fixed points $Q_{\mathrm{III}}, P_{\mathrm{III}}$ emerges left of the intersection of F_2 with $[0,1] \times \{1\}$—we call this intersection point $(\tilde{x}_1, 1)$. This implies that as β_1 is further increased the point P_{III} wanders through $(\tilde{x}_1, 1)$. Since P_{III} is stable along $x_2 = 1$ we always have $\partial \pi_1(p_{\mathrm{III}1}, p_{\mathrm{III}2})/\partial x_1 < 0$. In particular, this means that the pair of equilibria appears left of $(\tilde{x}_1, 1)$ if and only if

$$\frac{\partial \pi_1(\tilde{x}_1, 1)}{\partial x_1} = -B + \frac{C\tilde{\beta}_1}{(1+\gamma^{\mathrm{I}}+\tilde{\beta}_1\tilde{x}_1)^2} < 0, \tag{A.3}$$

where

$$A - B(1+\tilde{x}_1) - \frac{C}{1+\beta^{\mathrm{I}}+\gamma^{\mathrm{I}}\tilde{x}_1} = 0$$

and

$$\tilde{\beta}_1\tilde{x}_1 + \gamma^{\mathrm{I}} = \beta^{\mathrm{I}} + \gamma^{\mathrm{I}}\tilde{x}_1.$$

The same argument shows that the pair of equilibria created by increasing γ_1 appears left of $(\tilde{x}_1, 1)$ if and only if

$$\frac{\partial \pi_1(\tilde{x}_1, 1)}{\partial x_1} = -B + \frac{C\beta^{\mathrm{I}}}{(1+\tilde{\gamma}_1+\beta^{\mathrm{I}}\tilde{x}_1)^2} < 0,$$

where $\tilde{x}_1$ is defined as above and $\tilde{\gamma}_1$ by

$$\beta^{\mathrm{I}}\tilde{x}_1 + \tilde{\gamma}_1 = \beta^{\mathrm{I}} + \gamma^{\mathrm{I}}\tilde{x}_1.$$

Assuming that (A.3) holds we get

$$\begin{aligned}\frac{\partial \pi_1(\tilde{x}_1, 1)}{\partial x_1} &= -B + \frac{C\beta^{\mathrm{I}}}{(1+\tilde{\gamma}_1+\beta^{\mathrm{I}}\tilde{x}_1)^2}\\ &= -B + \frac{C\beta^{\mathrm{I}}}{(1+\beta^{\mathrm{I}}+\gamma^{\mathrm{I}}\tilde{x}_1)^2}\\ &< -B + \frac{C\tilde{\beta}_1}{(1+\gamma^{\mathrm{I}}+\tilde{\beta}_1\tilde{x}_1)^2}\\ &< 0.\end{aligned}$$

Hence, given that the equilibria created by increasing β_1 are stable in the transversal direction, also the equilibria created by increasing γ_1 have to be stable in the transversal direction. □

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