Does the "uptick rule" stabilize the stock market?
Insights from adaptive rational equilibrium dynamics

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Short selling regulation

- **Short selling**: the practice of selling financial instruments that have been borrowed, typically from a broker-dealer, with the intent to buy the same class of financial instruments in a future period and return them back at the time of maturity of the loan.

- Short sales can be used:
  1. to hedge the risk of long positions;
  2. to profit from an expected downward price movement.

- This financial practice may have negative effects on the stability of the stock markets.

- For this reason, many national authorities have developed different kinds of short selling restrictions. Most of them are ”price test based”, such as the ”Uptick rule”.
Mispricing due to a **reduction in stock supply** caused by short selling restriction (see, e.g. Miller, 1977).

Theoretical models with heterogeneous agents and differences in trading strategies support the idea that share values are overvalued under short selling restriction due to the fact that "pessimistic" traders are ruled out of the market, see, e.g., (Harrison & Kreps, 1978);

In contrast, theoretical models based on the assumption that all agents have rational expectations suggest that short selling restriction does not cause stock prices to be biased on average (see, e.g. Diamond & Verrecchia, 1987).

*The evidence supports the models with differences in beliefs rather than the rational expectations alternative,* see (Boehmer et al. 2008).
Rule 10a-1, or Uptick rule, imposed by the S.E.C. (in force until 2007): *short selling is allowed only on upward market movements.*

Three objectives:

(i) *allowing relatively unrestricted short selling in an advancing market;*

(ii) *preventing short selling at successively lower prices, thus eliminating short selling as a tool for driving the market down; and*

(iii) *preventing short sellers from accelerating a declining market by exhausting all remaining bids at one price level, causing successively lower prices to be established by long sellers.*
Empirical evidence of the effects of the uptick rule

- The last two objectives of the regulation have been confirmed by empirical analysis (see Gordon and Peterson, 1999).
- The regulation seems not effective in producing the first desired goal.

This work tries to provide some insights about the effects of the "uptick rule" by mean of a ARED asset pricing model as in Brock and Hommes (1998).

Other forms of short selling restrictions have been analyzed using the same simple asset pricing model in:

1. Dercole and Cecchetto (2010): investigated the complete banning of short selling;
An asset pricing model as in Brock & Hommes (1998) with positive supply of shares:

1. A *risk-free asset* with gross return \( R \) and a *risky asset* exchanged at price \( p_t \) and with IID dividend process \( (y_t)_{t \geq 0} \) with \( E(y_t) = \bar{y} \). Supply shares \( S > 0 \);

2. \( N \) agents with different beliefs:

\[
E[p_{t+1}] = E[p] + f_h(x_{t-1}, \ldots, p_{t-n}); \ h = 1, 2, \ldots
\]

3. Excess return per share in \( (t, t+1) \) is denoted by \( R_{t+1} = p_{t+1} + y_{t+1} - Rp_t \);

4. Mean-variance demand for the risky asset:

\[
z_{h,t} = \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})}
\]
1. **Market clearing mechanism:** (in deviations from the fundamental value);

\[ x_t = \frac{1}{R} \sum_h n_{h,t} f_h(x_t) \]

2. **Exponential replicator dynamic:**

\[ n_{h,t} = \frac{e^{\beta(\pi_{h,t}) - C_h}}{\sum_i e^{\beta(\pi_{i,t}) - C_i}}, \quad h = 1, 2, ... \] (1)

where \( \pi_{h,t} = R_t z_{h,t-1} \) is the excess return (or realized profit) and \( \beta \) is the *intensity of choice*.

3. In case of two types of belief:

\[ m_t = \tanh \left( \frac{\beta}{2} \left[ (\pi_{1,t} - \pi_{2,t}) - (C_1 - C_2) \right] \right), \quad m_t = n_{1,t} - n_{2,t}; \]
The ARED asset pricing model constrained by the uptick rule

Demand constraint under uptick rule:

\[
\begin{align*}
  z_{h,t} &= \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})} \\
  z_{h,t} &= \text{Max} \left\{ \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})}, 0 \right\}
\end{align*}
\]

if \((\phi x_{t-1} - x_{t-2}) > 0\)

if \((\phi x_{t-1} - x_{t-2}) < 0\)
The asset pricing model with short selling constraint and two types of belief:

**Market clearing:** \( s = \frac{S}{N} \)

- \( Z_0 \cup U: \sum_{i=1,2} \eta_i,t z_i,t = s \)
- \( Z_1: \eta_2,t z_2,t = s \) & \( z_1,t = 0 \)
- \( Z_2: \eta_1,t z_1,t = s \) & \( z_2,t = 0 \)
A nonlinear predictor: Smoothed-ROC

The Rate Of Change, see Elder (1992):

\[
ROC = \left( \frac{p_{t-1}}{p_{t-L}} \right)^{\frac{1}{L-1}} = \left( \frac{\bar{p} + x_{t-1}}{\bar{p} + x_{t-L}} \right)^{\frac{1}{L-1}}, \quad L \geq 2,
\]

The **ROC-predictor**

\[
E [p_{t+1}] = p_{t-1}ROC^2, \quad f (x_t) = E [p_{t+1}] - \bar{p}
\]

The smoothed-ROC (S-ROC):

\[
E [p_{t+1}] = p_{t-1} (\alpha_{ROC} ROC + (1 - \alpha_{ROC}))^2, \quad f (x_t) = E [p_{t+1}] - \bar{p}
\]

where:

\[
\alpha_{ROC} = \frac{2}{ROC^\alpha + ROC^{-\alpha}}
\]
Agents’ Beliefs

Heterogeneous expectations:

Set of belief types 1:

\[
\begin{align*}
E_{1,t} [p_{t+1}] &= \bar{p} \\
E_{2,t} [p_{t+1}] &= g x_t + \bar{p}
\end{align*}
\]

fundamentalist

chartist

Set of belief types 2:

\[
\begin{align*}
E_{1,t} [p_{t+1}] &= \bar{p} \\
E_{2,t} [p_{t+1}] &= p_{t-1} \left( \alpha_{ROC} ROC + (1 - \alpha_{ROC}) \right)^2
\end{align*}
\]

fundamentalist

(S-ROC)trader
B-H model with positive supply of shares

Figure: Top-Left Panel: $S = 0$, Top-Right Panel $S = 0.1$, Bottom-Left Panel $S = 0.2$ and Bottom-Right Panel $S = 0.3$. 
B-H model with and without uptick rule

Figure: Fundamentalists-Chartists: $R = 1.1$, $a = 1$, $\sigma = 1$, $\bar{y} = 1$, $\alpha = 10$, $S = 0.1$, $C = 1$. 
Figure: Fundamentalists-Chartists: $x_t > 0$ time series: $\beta = 3$, $R = 1.1$, $a = 1$, $\sigma = 1$, $\bar{y} = 1$, $g = 1.2$, $S = 0.1$, $C = 1$. 
Numerical results

Figure: Fundamentalists-Chartists: $x_t < 0$ time series: $\beta = 4$, $R = 1.1$, $a = 1$, $\sigma = 1$, $\bar{y} = 1$, $g = 1.2$, $S = 0.1$, $C = 1$. 
Numerical results

Figure: Fundamentalists-(S-ROC) traders: $R = 1.1, a = 1, \sigma = 1, \bar{y} = 1, \alpha = 10, S = 0.1, C = 1.$
Numerical results

Figure: Fundamentalists-(S-ROC) traders: (time series) $\beta = 1.4$, $R = 1.1$, $a = 1$, $\sigma = 1$, $\bar{y} = 1$, $\alpha = 10$, $S = 0.1$, $C = 1$. 
A summary of the effects of the uptick rule

- The stability of the fundamental equilibrium is not effected;
- The constrain price is higher than the unconstrained price;
- For $\beta > \overline{\beta}$, there is no effect on price fluctuations;
- For ($x_t < 0$) and $\beta \in [\underline{\beta}, \overline{\beta}]$, we observe an increase of the fundamental-revert movements for prices;
- The fractions $\eta_{1,t}$ and $\eta_{2,t}$ change due to the short selling restriction, $\eta_{1,t}$ increases;
- Multiple attractors are observed when $x_t > 0$ for the first set of belief types;
- Price fluctuation reductions are observable for some values of $\beta$ for the second set of belief types;
- The second objective of the regulation is clearly realized, the first is not observable under market clearing mechanism.
Conclusions and further researches

- Do short sellers on average behave as momentum traders or as contrarians?
- Under Uptick rule short sellers are more contrarian, see Boehemer et al. 2008. It would be interesting to study this phenomena using the same model with three trading strategies (fundamentalists, contrarians and trend followers).
- The effect of $\phi$?
- A short selling restriction not based on price variation test but on the level of demand and offer of shares.


