Intellectual Property Rights and Market Production of Knowledge in an Endogenous Recombinant Growth Model

Carla Marchese, Simone Marsiglio, Fabio Privileggi, Giovanni Ramello

Institute POLIS, DiGSPES – Università del Piemonte Orientale, Alessandria (Italy)

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Our model is based on the Tsur and Zemel (2007) setting – in which knowledge evolves according to the Weitzman (1998) recombinant expansion process – but here we assume that knowledge is produced in a decentralized IPR-based market rather than through direct supervision by a social planner.

The social planner decides how many resources to convey to buy new knowledge, which is immediately rendered available for free.

*Intellectual Property Rights (IPR)* in the form of patents are sold to the Government immediately after new knowledge has been produced, at a price equal to the (perceived) marginal cost.
Unlike standard growth model:
- R&D firms exploit (seed) ideas stemming for free from social non-market interactions
- R&D market is a *bilateral monopoly* (unique holder of the patent versus the government) and R&D profits are dissipated

Besides the *market failure* due to the *public good* nature of knowledge, IPR adds one more drawback: R&D firms do not account for the positive externalities of their activity within the recombinant process, make their choices accounting for increasing costs and earn *strictly positive instantaneous profits*, thus introducing another *market failure*

The latter market failure can be partially corrected by the Government through a *tax on such profits* whose proceeds are employed as a *subsidy* to purchase more new knowledge
Main results

1. For each given stock of knowledge, the *social unit cost of knowledge growth* for our decentralized economy is strictly larger than its analogous in the *first-best economy* of Tsur and Zemel (2007).

2. However, if growth conditions are satisfied, *in the long-run* our decentralized economy grows endogenously *along the same balanced growth path* as in the first-best economy of Tsur and Zemel (2007).

3. Applying a *Projection method* algorithm (Privileggi, 2010, 2011) and ordinary *Runge-Kutta* type algorithms to numerically simulate the optimal policy and the optimal transitory time-path trajectories of the main variables, we can compare *total welfare under different fiscal policy scenarios* – i.e., different tax/subsidy rates on R&D profits.

4. Our simulations show that total welfare is increasing in the tax/subsidy rate on R&D profits.
Weitzman (1998) assumes that new knowledge is produced by combining $m$ existing seed ideas: if such matching yields a new successful idea, it will be added to the stock of existing (seed) ideas to be recombinated again, and so on.

We exploit the Tsur and Zemel (2007) setting, in which the knowledge law of motion turns out to be

\[
\dot{A}(t) = \frac{J(t)}{\varphi[A(t)]},
\]

- $A(t)$ is the stock of knowledge (the total number of ideas) at time $t$
- $\dot{A}(t)$ is its time-derivative
- $J(t)$ is a measure of physical resources employed in matching ideas
- $\varphi[A(t)]$ is the expected unit cost of knowledge production
The centralized economy

The expected unit cost of knowledge production and Pareto optimality

- The unit cost of knowledge production, $\varphi(A)$, is a decreasing function of the stock of knowledge $A$ that depends on:
  1. the number of existing seed ideas, $H$
  2. the number of combinations of seed ideas, $m$, and
  3. the probability $\pi(\cdot)$ of obtaining a successful idea from each matching, itself depending on the ratio of physical resources employed, $J$, to existing seed ideas, $H$

- A fully rational and perfectly informed benevolent ‘regulator’, by taxing household’s income to finance knowledge production, can Pareto-optimally choose $J$ in order to foster knowledge growth according to a long-term perspective.
The economy is composed of the following actors:

1. households
2. firms producing a composite final good
3. R&D firms
4. the government

A benevolent dictator still decides how many resources to convey to new knowledge production, but now she buys knowledge from the R&D firm owing the patent.

Once $\hat{A}$ is purchased by the government, both $\hat{A}$ and the resulting seed ideas, $H$, become immediately free.

R&D firms combine $H$ with $J$ to produce new knowledge, $\hat{A}$, to be sold to the government; they face a two-stage decision problem:

1. decide whether invest into a patent (entails registration and legal costs)
2. in case of investment, decide how much $\hat{A}$ to produce
Many firms may compete for acquiring a patent.

Equilibrium under free entry arises if registration/legal expenses equal the profits of the holder of the patent; thus, all R&D profits are dissipated (Scotchmer, 2004, Fischer and Henkel, 2011).

In order to prevent the unique supplier to reap a monopolistic profit, the Government hides the amount of public expenditure available for buying new knowledge and asks the supplier to provide a schedule specifying how much $A$ it would supply for each price in a given set (Niskanen, 1971; Miller and Moe, 1983; Mueller, 1989).

Hence, the R&D firm takes the price, $\psi (A)$, of new knowledge $A$ as given when maximizes instantaneous profit.

Also factor $H$ is given: it is free (arises out of a social sharing non-market process) and the R&D firm does not take into account the externality on the seed formation stemming from its activity.
Thus, instantaneous profits are maximized only with respect to $J$

**FOC** with respect to $J$ plus the *market clearing* condition, $\psi(A) \dot{A} = G$, equating R&D firm’s revenues to the amount of resources, $G$, paid by the regulator for purchasing the new knowledge, yield the following *decentralized knowledge dynamics*:

$$\dot{A}(t) = \frac{G(t)}{\psi[A(t)]}$$

The *price of knowledge production*, $\psi(A)$, can be compared to the *unit cost of knowledge production* for the centralized model, $\varphi(A)$.
Proposition

1. The unit cost of knowledge production for the economy with decentralized R&D production is always larger than that for the centralized economy, $\psi (A) > \varphi (A)$, for all $A < \infty$.

2. However, for large $A$ they converge to the same value:
\[
\lim_{A \to \infty} \psi (A) = \lim_{A \to \infty} \varphi (A) = 1/\pi' (0)
\]

3. The instantaneous optimal profit of R&D firms is strictly positive for all $A < \infty$.

- Point 3 states that in this model the combination of factors $H$ and $J$ is not efficient, as a positive profit arises from the difference between the marginal production cost as perceived by firms and the actual marginal cost (equal to the average cost).
- This drawback tends to fade away in mature economies where $H$ is already abundant.
To implement efficiency as under centralization the government may introduce a subsidy, \( d \), so that \( (G + d) / \psi(A) = J/\varphi(A) = \dot{A} \).

Assuming that taxes can be levied on R&D firms’ profits at a rate \( 0 \leq \tau \leq 1 \), the subsidy \( d \) can be set equal to such proceedings, and the knowledge dynamics in our model is given by:

\[
\dot{A}(t) = \frac{G(t)}{\phi_\tau[A(t)\}] = \frac{G(t)}{\tau \varphi(A) + (1 - \tau) \psi(A)}
\]

Now the unit cost of knowledge production, \( \phi_\tau(A) \), is a convex linear combination of the unit costs under centralization and under decentralization, depending on the tax rate \( \tau \).

- Setting \( \tau = 1 \), the government would be able to replicate the same first-best knowledge growth path under centralization.
- \( \tau = 0 \) implies fully decentralized, inefficient knowledge production.

We aim at studying how the economy reacts to different tax policies – different (constant) values of \( 0 \leq \tau < 1 \).
Model specification I

Assumption

1. \( m = 2 \) and the success probability function is hyperbolic: 
\[
\pi(x) = \frac{\beta x}{(\beta x + 1)}, \quad (\beta > 0 \text{ efficiency parameter})
\]

2. Labour is constant through time and normalized: \( L \equiv 1 \)

3. Cobb-Douglas production depends on capital, \( k \), and (labour-augmented) knowledge: 
\[
F(k, A) = \theta k^\alpha A^{1-\alpha}, \quad \theta > 0, \quad 0 < \alpha < 1,
\]
identical output producing firms act in a competitive market

4. CIES instantaneous utility: 
\[
u(c) = \frac{(c^{1-\sigma} - 1)}{(1 - \sigma)}, \text{ with } \sigma \geq 1
\]
Given the tax and discount rates, $0 \leq \tau < 1$ and $\rho > 0$, the social planner levies $G$ as a *tax on the representative consumer* and solves

$$V(k_0, A_0) = \max_{\{c(t), G(t)\}} \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt$$

subject to

\[
\begin{align*}
\dot{A}(t) &= G(t) / \phi \tau [A(t)] \\
\dot{k}(t) &= \theta k(t)^\alpha A(t)^{1-\alpha} - G(t) - c(t),
\end{align*}
\]

with the additional constraints $G(t) \leq y(t)$, $c(t) \leq k(t) + y(t)$, and usual non-negativity constraints.
On the locus $F_k(k, A) - F_A(k, A) / \phi_\tau(A) = 0$ on the state space $(A, k)$ the value of the marginal product of capital equals that of knowledge. It can be written as a function of $A$, $\tilde{k}_\tau(A)$, which we call the (transitory) turnpike:

for $A \to \infty \tilde{k}_\tau(A)$ becomes an affine function,

$\tilde{k}_\infty(A) = \alpha (A + 2 - \tau) / [\beta (1 - \alpha)]$, which we call the asymptotic turnpike. $\tilde{k}_\tau(A) > \tilde{k}_\infty(A)$ for all $A < \infty$, and $\tilde{k}_\tau(A)$ approaches $\tilde{k}_\infty(A)$ from above as $A$ increases.

The locus $F_k(k, A) = \rho$ defines the (linear) stagnation line

$\hat{k}(A) = (\theta \alpha / \rho)^{1/(1-\alpha)} A$.
Proposition

If the stagnation line lies above the asymptotic turnpike, i.e., if

$$\rho < r_\infty = \theta \alpha \left[ \beta (1 - \alpha) / \alpha \right]^{1-\alpha},$$

and the initial physical capital $k_0$ is sufficiently high with respect to the initial knowledge stock $A_0$, i.e. if $k_0 \geq k^{sk}_T (A_0)$ where $k^{sk}_T (A_0)$ is a Skiba-type point, then the economy grows along the transitory turnpike path, $\tilde{k}_T [A(t)]$, which, in the long-run, converges to a balanced growth path along the asymptotic turnpike, $\tilde{k}_\infty [A(t)]$, in which knowledge, capital, output, etc., all grow at the same constant rate:

$$\gamma = (r_\infty - \rho) / \sigma$$
Proposition

1. Turnpikes are monotonic with respect to the tax rate $\tau$ for fixed $A$:
   \[ \tau_1 < \tau_2 \iff \tilde{k}_{\tau_1}(A) > \tilde{k}_{\tau_2}(A) \text{ and } \tilde{k}_{\tau_1}^{\infty}(A) > \tilde{k}_{\tau_2}^{\infty}(A) \]

2. moreover, $\tau_1 < \tau_2 \iff y_{\tau_1}(A) > y_{\tau_2}(A) \text{ and } r_{\tau_1}(A) < r_{\tau_2}(A)$

Corollary

Economies with less public intervention – smaller $\tau$ – require larger threshold capital stock for growth to start.

Therefore, the IPR system implies a difference in opportunities available to different countries according to their initial conditions, allowing an easy takeoff only to the richest ones.

The taxation/subsidy scheme mitigates these effects bringing down the price of knowledge at a level more close to the social marginal cost.
When the growth conditions $\rho < r_\infty$ and $k_0 \geq k_t^{sk}(A_0)$ are met, the long-run equilibrium is the same for all $0 \leq \tau < 1$; hence, we must focus on transition dynamics – which do depend on $\tau$ – in order to perform welfare comparisons.

Under growth conditions there are two types of transitions:

1. one driving the system toward the turnpike starting from outside it
2. one describing the path along the turnpike after it has been entered

When growth conditions are not satisfied the transition is toward one point on the stagnation line.
Along the (transitory) turnpike the optimal trajectory is numerically approximated through the Projection method built by Privileggi (2010, 2011).

We assume that the initial capital, \( k_0 \), lies above or on the turnpike.

If \( k_0 \) lies strictly above the turnpike, \( k_0 > \tilde{k}_\tau (A_0) \), the trajectory from \((A_0, k_0)\) in \( t = 0 \) to a point \((A_r, \tilde{k}_\tau (A_r))\) on the turnpike at \( t = t_0 > 0 \) is computed through a shooting procedure which uses Runge-Kutta type approximations of the solution of the system of ODEs describing optimal necessary conditions on the Hamiltonian.

The value \( A_r \) is being reached in different instants depending on whether coming from above or along the turnpike.

Specifically, the system runs much faster when it starts above the turnpike rather than when it starts on the turnpike in \( t = 0 \).

This is because above the turnpike it is optimal to invest all output in R&D, \( G = F (k, A) \), while a fraction of capital is being consumed.
Whole transition trajectories and our parameterization

- We build the whole time-path trajectories from $t = 0$ as piecewise functions by joining each trajectory above the turnpike with the trajectory along the turnpike on $t = t_0$ on the point $A(t_0) = A_r$

- We assume the following fundamentals in our economy:

  $$\alpha = 0.5, \quad \rho = 0.04, \quad \theta = 1, \quad \sigma = 1 \text{ (log utility)}, \quad \beta = 0.01429$$

- three different tax/subsidy policies:

  $$\tau_L = 0 \text{ (full decentralization)}, \quad \tau_M = 0.4, \quad \tau_U = 0.8$$

- same initial stock of knowledge and physical capital in all 3 scenarios:

  $$A_0 = 2.33, \quad k_0 = 792.34$$

- $k_0$ lies on the ‘highest’ turnpike among the three, $k_0 = \tilde{k}_L(A_0)$, corresponding to full decentralization
On the left see:

- the *stagnation line* \( \hat{k}(A) \) [in red]

\[ k_{0L} \]

\[ \hat{k}(A) \]
Stagnation line and turnpikes

- On the left see:
  - the **stagnation line** $\hat{k}(A)$ [in red]

- **turnpikes**:
  - $\tilde{k}_L(A)$ 
    ($\tau_L = 0$) starting from 
    $k_0 = k_{0L} = \tilde{k}_L(A_0) = 792.34$ [in light grey]
On the left see:

- the stagnation line \( \hat{k}(A) \) [in red]

**turnpikes:**

- \( \tilde{k}_L(A) \) \((\tau_L = 0)\) starting from \( k_0 = k_{0L} = \tilde{k}_L(A_0) = 792.34 \) [in light grey]
- \( \tilde{k}_M(A) \) \((\tau_M = 0.4)\) starting from \( k_{0M} = \tilde{k}_M(A_0) < 792.34 \) [in dark grey] and
On the left see:

- the *stagnation line* $\hat{k}(A)$ [in red]

**turnpikes:**

- $\tilde{k}_L(A)$ ($\tau_L = 0$) starting from $k_0 = k_{0L} = \tilde{k}_L(A_0) = 792.34$ [in light grey]
- $\tilde{k}_M(A)$ ($\tau_M = 0.4$) starting from $k_{0M} = \tilde{k}_M(A_0) < 792.34$ [in dark grey] and
- $\tilde{k}_U(A)$ ($\tau_U = 0.8$) starting from $k_{0U} = \tilde{k}_U(A_0) < \tilde{k}_M(A_0) < 792.34$ [in black]
For $\tau_M = 0.4$ and $\tau_U = 0.8$ the initial capital $k_0 = \tilde{k}_L (A_0) = 792.34$ lies above both turnpikes $\tilde{k}_M (A)$ and $\tilde{k}_U (A)$ in $t = 0$.

Through *backward shooting* get two values for the stock of knowledge on which optimal capital and consumption, as functions of $A$, intersect their respective turnpike: $A_{rM} = 4.54$ and $A_{rU} = 5.36$.
Remark

Because it is optimal to invest all output in R&D when \( k \) lies above the turnpike, for both \( \tilde{k}_M (A) \) and \( \tilde{k}_U (A) \) the time period required to reach \( A_r \) starting from above the turnpike turns out to be ten times shorter than the time period necessary to reach the same level \( A_r \) starting already on the turnpike, along which it is optimal to invest only a fraction of the output in R&D.

This clearly deeply affects total welfare, as our simulation will show.
Below see the whole time-path trajectories of $A(t)$, $k(t)$, $y(t)$ and $c(t)$ in *all three tax/subsidy policy* scenarios, $\tau_L = 0$ (full decentralization), $\tau_M = 0.4$ and $\tau_U = 0.8$ all starting from *initial values* $A_0 = 2.33$ and $k_0 = 792.34$ in $t = 0$.
Whole optimal time-path trajectories II
Output and (optimal) consumption
Whole optimal time-path trajectory III

Optimal R&D financing

\[ \text{Diagram showing whole optimal time-path trajectory III.} \]
Welfare analysis

To evaluate total welfare we approximate the integral
\[ \int_{0}^{\infty} \ln [c(t)] e^{-0.04t} dt \]
along the whole optimal consumption trajectories \( c(t) \), including the trajectory leading to stagnation in order to check the Skiba-type condition (using a Gauss-Legendre quadrature routine on 2000 nodes for \( 0 \leq t \leq 400 \), analytically either on the steady state or along steady growth for \( t > 400 \))

We found that:

1. under no tax/subsidy policy (\( \tau_L = 0 \), full decentralization) the economy ends up in stagnation, with total welfare = 94.74
2. under a ‘middle’ tax/subsidy policy (\( \tau_M = 0.4 \)) the economy grows along a BGP in the long-run, with total welfare = 95.98
3. under a ‘strong’ tax/subsidy policy (\( \tau_U = 0.8 \)) the economy grows along a BGP in the long-run, with total welfare = 97.87

Such findings suggest that total welfare is increasing in the tax/subsidy rate \( \tau \)
Conclusions
Our approach and our findings

- To perform welfare analysis in our decentralized setting we had to rely on transition dynamics.
- ... tough objects when knowledge is produced according to the Weitzman (1998) recombinant process.
- Only by numerical approximation of the solution of a suitable detrendization of the model we were able to compute the optimal time-path trajectories.
- Our simulation shows that total welfare increases in the tax/subsidy policy, that is, larger values of $\tau$ yield larger total welfare.
- We found that this monotonicity property is robust.
- For our choice of parameters’ values and initial conditions a no tax/subsidy policy ($\tau_L = 0$) actually leads the economy toward stagnation while public intervention can boost sustained growth.