A reconsideration of the formal Minskyan analysis: microfoundations, endogenous money and the public sector

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Introduction

• The issues in modelling the Financial Instability Hypothesis

• The baseline model (Chiarella and Di Guilmi, JEDC 2011) with analytical solution

• The extension with government (Chiarella and Di Guilmi, SNDE 2012) and monetary policy

• Variables are written with the superscript $j$ when they refer to a generic firm, with the subscript $z(= 1, 2)$ when referring to a microstate, and without any sub- or superscript when indicating aggregate values
1 **Issue 1: can we model the FIH?**

- **The risk of oversimplification (Foley, 2001):**
  - “It is not easy to formulate a single, generic, range of assets to represent the multifarious vehicles for the financial maneuvers that lie behind financial fragility”;
  - “the formal, statistical methods adopted by contemporary economists are inherently hostile to critical and qualitative insights into the performance of markets as human and social institutions”.

- **A tentative response:**
  - an endogenous mechanism of creation of credit within a framework which can embody market sentiment feedback;
  - an analytical solution which allows for an evolving state space.
  - A quantitative approach can foster interaction with the rest of the profession.
2 Issue 2: how to model heterogeneous and interacting agents?

- **Relevance of the micro-analysis in the FIH:**
  - “an ultimate reality in a capitalist economy is the set of interrelated balance sheets among the various units” (Minsky, 1975);
  - “Shifts of firms among classes as the economy evolves in historical time underlie much of its cyclical behavior. This detail is rich and illuminating but beyond the reach of mere algebra” (Taylor and O’Connell, QJE 1985).

- **Two different methods for model solution:**
  1. the **agent based model** with numerical simulation;
3 Issue 3: endogenous money and the public sector (Lavoie wp 2008)

- Endogenous money: the quantity of liquidity is endogenously determined to balance a Tobinian asset portfolio system;
- Big government and central bank (for the moment with ABM solution).
3.1 The context

- Minsky (1975): $I_t = f(P_{k,t} - P_t)$;
- Taylor and O’Connell (QJE1985) and Franke and Semmler (1989):
  - $P_k = g(\rho)$
  - $\rho$ is the expected return to capital for the economy and influences the demand for equities.
- Our treatment:
  1. microfoundation: $\rho^j$ is the expected return to capital for the firm $j$:

$$P_k^j(t) = \frac{\rho^j(t)P(t)}{i(t)} \quad (1)$$

  where $i$ is the interest rate, $P$ is the final good price.
  2. $\rho^j$ is endogenous: dependent on the dominant strategy in the financial market.
4 Private sector model

4.1 Hypotheses

**Firms**

- A firm $j$ decides on investment based on the shadow-price of capital $P^j_k(t)$:

  \[ I^j(t) = a \left[ P^j_k(t) - P(t) \right] \tag{2} \]

  and then accordingly about workforce and output;

- Firms prefer to finance their production costs:
  - first with retained earnings $A^j$ and, then
  - with new equities $E^j$ or debt $D^j$ (in a proportion dependent on the level of interest rate);
Firms are classified into two groups according to their level of debt $D^j$: 

- **speculative firms**: $D^j(t) > 0$
- **hedge firms**: $D^j(t) = 0$

Correspondingly, there are *two types of shares* in the market, with prices $P_{e,1}$ and $P_{e,2}$.

A firm fails if $D^j(t) > cK^j(t)$, with $c > 1$; it is replaced with a probability directly proportional to the variation in the aggregate output observed in the previous period.

Salaries vary to match production and demand.
Investors

- Two possible types of investors: *chartists* (proportion $n^c$) and *fundamentalists* (proportion $1 - n^c$);
- we assume that chartists *on average* favour the speculative firms:

$$\rho_1^j(t) = \tilde{u}^j(t) n^c(t), \quad \rho_2^j(t) = \tilde{u}^j(t) (1 - n^c(t));$$

where

- $\tilde{u}^j$ is an idiosyncratic random variable with $\mathbb{E}[\tilde{u}^j] = 1$;
- the proportion of chartists in the market $n^c$ is randomly drawn from a uniform distribution.
Wealth allocation

- using the mean field approximations $\rho_1$ and $\rho_2$ (the means of $\rho^1_z, \ldots, \rho^j_z, \ldots, \rho^N_z$), prices and allocations of the wealth $W$ are calculated according to

\[
\begin{align*}
\epsilon_1(i, \rho_1, \rho_2, \psi)W &= P_{e,1}E_1 \\
\epsilon_2(i, \rho_1, \rho_2, \psi)W &= P_{e,2}E_2 \\
\beta(i, \rho_1, \rho_2, \psi)W &= D \\
\Psi(i, \rho_1, \rho_2, \psi)W &= M \\
W &= P_{e,1}E_1 + P_{e,2}E_2 + D + M
\end{align*}
\]

where:

- the parameter $\psi$ reflects the capacity of the system to generate endogenous money;
- $M$ the demand for liquid assets, $D$ the debt and $E_z$ are the quantity of shares.
The variable $\rho$

- The key variable for the allocation of wealth is $\rho^j$. It influences:
  - the level of firms’ **investment** through the shadow price
    \[
    P_k^j(t) = \frac{(r(t) + \rho^j(t))P}{i(t)};
    \]
  - the prices of shares $P_{e,1}$ and $P_{e,2}$ in system (3), reflecting the investors’ expectations on the different firms.
4.2 Stochastic dynamics

The two dynamics

• Using the mean field approximations $\rho_1$ and $\rho_2$ it is possible to replicate the model for a representative hedge firm and for a representative speculative firm;

• thus the model is able to generate dynamics in two different ways:
  – an agent based approach with $N$ different agents;
  – a stochastic approximation, with 2 different firms: one “good” and one “stressed”.
The method: stochastic dynamic aggregation

How to aggregate *heterogeneous* and *evolving* agents?

1. Agents are classified into different *micro-states*, according to their characteristics;

2. A *representative* agent for each cluster is identified (*Mean-field approximation*);

3. Macro configuration is identified by the *number of agents that occupy each micro-state at a given time* (the *macro-state*), governed by a *stochastic law*;

4. This stochastic law is functionally modelled as a *master equation* (ME).
• Endogenous transition rates $\lambda(t)$ and $\mu(t)$: estimated as functions of the shocks on $\rho$ (exogenous but with known probability).
• The stochastic dynamics of the proportion of the two types of firms can be described by a **master equation**:

\[
\frac{dp(N_1, t)}{dt} = \left( \text{inflows of probability fluxes into state 1} \right) - \left( \text{outflows of probability fluxes out of state 1} \right)
\]

\[
\frac{dp(N_1, t)}{dt} = \lambda p(N_1 - 1, t) + \mu p(N_1 + 1, t) - \left\{ [\lambda + \mu] p(N_1, t) \right\}
\]

(4)
Evaluating the components of the dynamics

- split the state variable $N^1$ in two components:
  - the **drift** ($m$): tendency value of the mean for $n^1 = N^1/N$;
  - the **spread** ($s$): aggregate fluctuations around the drift;
  - hypothesis:
    
    \[ N^1 := Nm + \sqrt{N}s \]  
    \( (5) \)
• **Solution:**

  – *Macroscopic equation* (the drift):

    \[
    \frac{dm}{dt} = \lambda m - (\lambda + \gamma)m^2
    \]  \hspace{1cm} (6)

  – Probability density of fluctuations:

    \[
    p(s) = C \exp \left( -\frac{s^2}{2\sigma^2} \right) \, : \, \sigma^2 = m^* \frac{\gamma}{\lambda + \gamma}
    \]  \hspace{1cm} (7)
An equation for aggregate investment

- Mean field approximation: a representative unit for each of the states
  - investment for each firm in the two groups: $I_1$ and $I_2$.
- Trend of aggregate investment

\[
I(t) = \int \left\{ I_1(t)n_1(t) + I_2(t)[1 - n_1(t)] \right\} \, dt
\]  \hspace{1cm} (8)
Using the asymptotic solution, the dynamics of the economy can be represented by the following system:

\[
\begin{align*}
    dn_1(t) &= (\lambda n_1(t) - (\lambda + \mu)[n_1(t)]^2)dt + \sigma \, d\theta \\
    dK(t) &= I(t) = N \left\{ I_1(t)n_1(t) + I_2(t)[1 - n_1(t)] \right\} dt
\end{align*}
\]

(9)

where

\[
\theta(s) \sim C \exp \left( -\frac{s^2}{2\sigma^2} \right) : \quad \sigma^2 = \frac{\lambda\mu}{(\lambda + \mu)^2}
\]

(10)

with

- \( n_1 \) indicates the proportion of speculative firms;
- \( s \) represents the fluctuating component of the stochastic process for \( n_1 \).
4.3 Simulations

Figure 1: Capital (upper panel) and share of speculative firms (lower panel). Agent based model (black continuous line) and stochastic dynamics (red dashed line).
Figure 2: Debt/capital ratio (left axes) and aggregate capital (right axis). Simulation of the agent based model.
Figure 3: Aggregate value of assets.
• The distribution of amplitudes similar to what is observed

• During the up-turn proportion of speculative firms grows, until the peak is reached. Then over-indebtedness generates a wave of bankruptcies.

• Counter-cyclical fiscal policy reduces the volatility of aggregate production.

• The most effective stabilization policies involve financial and bankruptcy regulations.
Figure 4: Upper panel: fits for speculative and hedge firms capital during expansions and recessions. Lower panel: Lognormal distribution fit of hedge firms capital at different time steps.
Figure 5: Aggregate capital, variance of fluctuations, interest rate and wealth for different values of ψ (Monte Carlo agent based simulation).
Figure 6: Aggregate capital, variance of fluctuations, interest rate and wealth for different values of $c$. 
5 Concluding remarks

5.1 Results

- Macro-behaviour determined by the change in the distribution of firms;
- Regulation (on the creation of endogenous credit and bankruptcy) can stabilise the system;
- Fiscal and monetary policy alone are not effective for stabilisation;
- A tax on wealth and an opportune monetary policy can eliminate the crowding-out;
- A high sensitivity to price of the CB reduces GDP volatility but increases financial instability.